Public School Arithmetic

PRICE 25 CENTS

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THE

PUBLIC SCHOOL

ARITHMETIC

AND

MENSURATION

BY

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(INSPECTOR OF PUBLIC SCHOOLS, OTTAWA.)

Authorized for use in the Public Schools of Ontario by the Minister of Education.

ENLARGED EDITION.

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Donald McQueen
Cedarville
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PREFACE.

In this edition of the Public School Arithmetic the original plan of the book has been enlarged to include the additional arithmetical work required for the recently established Public School Leaving Examination. The first eight chapters to Page 160 cover the arithmetic of the High School Entrance Examination, and *remain practically the same as they were in the first edition.* The chief differences are :—

1st. The solved examples and the accompanying explanations which were contained in Chapters IV. and V. of the former edition have been omitted from this edition. It is left to the teacher to exhibit these solutions on the blackboard, and to give all necessary explanations orally.

2nd. Exercises XXXIV. and XXXVIII. have been omitted.

3rd. A section on Insurance has been added to Chapter VIII., and the subjects of Interest and Bank Discount have been treated at somewhat greater length.

4th. The exercises have been set up in larger type, and in consequence a problem has occasionally been left out, or a new problem inserted to suit the requirements of the pages. *These changes have, however, been so few that no difficulty should be felt in using the two editions in the same school.*

Chapters IX., X. and XI., and the section on Stocks and Bonds in Chapter VIII. are new, and contain the additional work prescribed for the Fifth Form in the Public Schools. They are in great part drawn from the Author's "Arithmetic for High Schools," published by the Rose Publishing Company of Toronto, and to that work teachers who desire to see a fuller treatment of the subjects of these chapters are referred.

The number of problems for solution has been increased from 2,211 in the former edition to 2,555 in this edition.

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ARITHMETIC.

CHAPTER I.

OF NUMBERS AND NOTATION.

The ten marks or characters

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,

denoting *nought*, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine* respectively, are called **Arabic Numerals** or **Figures.** The first is called **nought**, *cipher* or *zero*. The remaining nine are called **digits.**

A number expressed in Arabic numerals is said to be written in **Arabic Notation.**

The letters

I, V, X, L, C, D, M,

denoting *one*, *five*, *ten*, *fifty*, *one hundred*, *five hundred*, *one thousand* respectively, are called **Roman Numerals.**

A number expressed in Roman numerals is said to be written in **Roman Notation.**

[No exercise in notation is given here, because practice therein should be intermingled with practice in addition and subtraction, and should advance concurrently therewith. It is left to the teacher to introduce in their proper places exercises suited to the progress of his pupils.

The following classes of problems may be proposed:—

1st. Expression in Arabic Notation of numbers presented objectively;

2nd. Objective presentation of numbers expressed in Arabic Notation;

3rd. Reading numbers written in Arabic Notation, and writing them in words;

4th. Expression in Arabic Notation of numbers written in words;

5th. Reading numbers written in Roman Notation, and writing them in words;

6th. Expression in Roman Notation of numbers written in words;

7th. Changing from Roman Notation to Arabic Notation, and *vice versa*.

Pupils, while practising these exercises, should be given, by means of objective representations, a clear insight into the fundamental principles of Arabic Notation.]

In counting objects and in measuring magnitudes the standard by which we count or we measure is called **a Unit.** Thus in counting the pupils in a class the unit is *one pupil;* in counting the pages in a book the unit is *one page;* in counting eggs by the dozen the unit is *one dozen eggs;* in selling bricks by the thousand the unit is *a thousand bricks;* in measuring cloth by the yard the unit of length is *one yard;* in weighing sugar by the pound the unit of weight is *one pound;* in measuring apples by the bushel the unit of volume is *one bushel.*

Every number is either abstract or concrete.

An Abstract number *is one that does not specify what the objects are that are counted, or of what kind the magnitude is that is measured.* Thus 4, 7, 12, 25 pairs, 9 dozen, 37 thousand, are abstract numbers. **An abstract number,** therefore, **signifies only the number of times some unit is repeated.**

A Concrete Number *is one that specifies not only the numerical value of the quantity, but also what the objects are that are counted, or of what kind the magnitude is that is measured.* Thus 4 boys, 7 books, 12 pencils, 25 pairs of skates, 9 dozen oranges, 37 thousand bricks, are concrete numbers. A concrete number is not, strictly speaking, a mere number, but is rather a CONCRETE QUANTITY; and its complete representation must consequently consist of two parts—the one representing the numerical value (the number proper), the other naming the things counted or the standard of measurement used.

Like Numbers are numbers that have the same unit.

Unlike Numbers are numbers that have different units.

[Pupils should be practised in the use of the above-defined terms till they understand them clearly and are thoroughly familiar with them. The test of sufficient knowledge is not the ability to repeat, however glibly, the definition of any term, but the unhesitating employment of each term wherever it ought to be used and nowhere else. Exercises should be given—

1st. In naming the units in proposed numbers;

2nd. In distinguishing the abstract from the concrete numbers in mixed groups of these;

3rd. In distinguishing groups of like numbers from groups of unlike numbers;

4th. In assorting into separate sets the several classes of like numbers contained in a miscellaneous group.]

CHAPTER II.

THE FOUR FUNDAMENTAL OPERATIONS.

I. ADDITION.

A number which as a whole is made up of two or more numbers as parts is called the **Sum** *of these numbers.*

Addition *is the operation by which we find the sum of two or more numbers.*

The numbers to be added together are called **Addends.**

The sign of addition is +, read *plus*. This sign + written before a number denotes that the number is an addend. Thus 4+3 is read "four plus three," and denotes that three is to be added to 4. Again, 63+22+13 is read "sixty-three plus twenty-two plus thirteen," and denotes that 22 is to be added to 63 and then 13 added to their sum.

Only like numbers can be added together. Unlike numbers cannot be added into a single sum, but give a separate sum for each distinct unit or standard.

[In this and in the following chapter two classes of exercises have been omitted from each section:—

1st. Simple problems in concrete numbers introductory to the several rules or methods of calculation;

2nd. Exercises intended for mere mechanical drill in computation, with the design to ensure accuracy and rapidity therein. At this stage of the learner's advancement in the art of reading and of his training in reasoning, all introductory problems should be presented *viva voce*, and before any reference is made to the text-book and the explanation given therein. These problems, to be of any value, must therefore be supplied by the teacher himself, and should be adapted in variety and number to the intellectual development of the pupils, and in their character and subject matter to the particular circumstances and the every-day experience of the class. Exercises in mechanical drill are so easily supplied that it is unnecessary to devote to them space needed for other classes of problems.]

ADDITION TABLE.

1	1	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	8	9
	1	2	3	4	5	6	7	8	9	10
2	2	2	2	2	2	2	2	2	2	2
	0	1	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9	10	11
3	3	3	3	3	3	3	3	3	3	3
	0	1	2	3	4	5	6	7	8	9
	3	4	5	6	7	8	9	10	11	12
4	4	4	4	4	4	4	4	4	4	4
	0	1	2	3	4	5	6	7	8	9
	4	5	6	7	8	9	10	11	12	13
5	5	5	5	5	5	5	5	5	5	5
	0	1	2	3	4	5	6	7	8	9
	5	6	7	8	9	10	11	12	13	14
6	6	6	6	6	6	6	6	6	6	6
	0	1	2	3	4	5	6	7	8	9
	6	7	8	9	10	11	12	13	14	15
7	7	7	7	7	7	7	7	7	7	7
	0	1	2	3	4	5	6	7	8	9
	7	8	9	10	11	12	13	14	15	16
8	8	8	8	8	8	8	8	8	8	8
	0	1	2	3	4	5	6	7	8	9
	8	9	10	11	12	13	14	15	16	17
9	9	9	9	9	9	9	9	9	9	9
	0	1	2	3	4	5	6	7	8	9
	9	10	11	12	13	14	15	16	17	18
10	10	10	10	10	10	10	10	10	10	10
	0	1	2	3	4	5	6	7	8	9
	10	11	12	13	14	15	16	17	18	19

EXERCISE I.

1. On the reopening of school after the midsummer holidays John bought a Third Reader for 36 cents, an arithmetic for 25 cents, a grammar for 25 cents, a geography for 68 cents, a slate for 15 cents, a copy book for 13 cents, and a bottle of ink, a pen, and a penholder for 8 cents. How much did the whole cost?

2. There were 339 pages in his Third Reader, 192 in his arithmetic, 154 in his grammar, and 96 in his geography. How many pages were there in all?

3. In the Third Reader there were 7248 words; in the arithmetic, 2179; in the grammar, 2396; and in the geography, 6947. How many words were there altogether in the four books?

4. In the Third Reader there were 27,496 letters; in the arithmetic, 9,876; in the grammar, 13,789; and in the geography, 34,874. How many letters were there altogether in the four books?

5. In the school there were 19 pupils in the first class, 15 in the second, 17 in the third, 9 in the fourth, and 6 in the fifth. How many pupils were there in the school?

6. John had 8 marbles, his uncle gave him 19, and on his way to school he bought 26. How many had he then?

7. The other boys in John's class were William, James, Edward, Thomas, Henry and George. How many letters were there in the name of each boy in the class, and how many letters in all their names together?

8. During the first week of school John received 27 merit marks, during the second week 29, during the third week 34, and during the fourth week 19. How many merit marks did he receive during the four weeks?

9. He received 109 merit marks in September, 137 in October, 128 in November, and 97 in December. How many did he receive during the whole four months?

10. Having been first on the honor roll for October, his father gave him 25 cents, his mother gave him 18 cents, his uncle gave him 15 cents, and his aunt gave him 7 cents. He put all into his savings bank, in which he already had 49 cents. How much had he then in his bank?

11. John attended school 16 days in January, 19 days in February, 22 days in March, 17 days in April, 22 days in May, and 19 days in June. How many days did he attend school during the six months?

12. The school room is 17 steps long and 14 steps wide. How many steps would go completely round it?

13. The school-yard fence has 397 pickets on the front, 254 on each end, and 578 on the back. How many pickets are there on the whole fence?

14. In a game of baseball one side made 17 runs the first innings, 14 runs the second innings, 19 runs the third innings, 8 runs the fourth innings, and 26 runs the fifth innings. The other side made in their corresponding innings 4 runs, 19 runs, 23 runs, 29 runs and 8 runs respectively. How many runs did each side make?

15. In the first innings in a game of cricket the first boy out made 23 runs; the second boy made 19 runs; the third, 7 runs; the fourth, 39 runs; the fifth, 19 runs; the sixth, 3 runs; the seventh, no runs; the eighth, 9 runs; the ninth, 7 runs; the tenth, 2 runs; and the last boy carried out his bat for 5 runs. There were 4 wides and 9 byes. What was the total of the innings?

16. In a game of cricket one bowler bowled 395 balls, a second bowled 259 balls, a third bowled 179 balls, a fourth bowled 198 balls, a fifth bowled 97 balls, and a sixth bowled 69 balls. How many balls were bowled in all?

17. Annie has 3 cents, a five-cent piece, a ten-cent piece, a twenty-five-cent piece, and a fifty-cent piece. How much has she in all?

18. Annie's hen has 9 chickens, Jane's has 8 chickens, Fannie's two have 19 chickens together, and Bertha's three have 27 chickens. How many hens and how many chickens are there altogether?

19. John costs his father $67 a year for food, $25 for lodging, $39 for clothes, $7 for pocket money, and $47 for other expenses. How much a year does he cost his father in all?

20. Thomas cost his parents $57 the first year of his life, $49 the second year, $63 the third year, $78 the fourth year, $69 the fifth year, $74 the sixth year, and $85 the seventh year. How much did he cost them the whole seven years?

21. Annie paid 18 cents for milk, 35 cents for cream, 48 cents for eggs, 37 cents for butter, and 26 cents for cheese. How much did she pay in all?

22. How many letters are there in the names of the days of the week?

23. How many letters in the names of the months?

24. Martha gathered 37 eggs on Monday, 29 on Tuesday, 18 on Wednesday, 24 on Thursday, and 15 on Friday. How many did she gather during the five days?

25. During the week of the fair Harry spent 34 cents for peaches, 28 cents for pears, 17 cents for apples, 24 cents for oranges, and 9 cents for candy, and had 13 cents left. How much did he spend and how much had he at first?

26. A man paid $19 for a suit of clothes, $15 for an overcoat, $3 for a hat, $4 for a pair of boots, $24 for underclothes, and $7 for other articles. How much did he pay for the whole?

27. A man spent $174 a year on clothing for his family, $369 for food, $168 for house-rent, $69 for fuel, $27 for light, $77 for furniture, $84 for wages, and $67 for incidentals; he also paid $18 to a doctor and $24 for taxes. How much a year did he spend on all these things together?

28. A merchant sold $278 worth of goods on Monday, $395 worth on Tuesday, $647 worth on Wednesday, $594 worth on Thursday, $295 worth on Friday, and $947 worth on Saturday. What was the total value of his week's sales?

29. Four men built and equipped a mill. The first paid on it $7418, the second $9475, the third $8643, and the fourth $7464. How much did the mill cost them?

30. A drover bought 78 sheep on Monday for $395, 49 sheep on Tuesday for $313, 36 sheep on Wednesday for $194, 57 sheep on Thursday for $328, 65 sheep on Friday for $347, and 193 sheep on Saturday for $978. How many sheep did he buy during the week, and how much did they cost him?

31. In the first car of a railway train there were 27 passengers, in the second car 36, in the third car 29, and in the drawing-room car 18. How many persons were on the train, counting in the conductor, the drawing-room car conductor, the driver, the fireman, the mail clerk, the express clerk, two brakesmen and the newsboy?

32. In a cattle train there were two cars with 17 head of cattle in each, three cars with 19 head in each, one car with 22 head, and two cars with 21 head in each. How many head were there in all?

33. A farmer had 27 acres of land under wheat, 15 acres under oats, 14 acres in meadow, 19 acres in pasture, 9 acres under peas, 6 acres in potatoes, 7 acres in turnips, 9 acres under Indian corn, 5 acres in orchard, 3 acres for house, garden, stables, barns and barn-yards, and 29 acres of woods. How many acres in his farm?

34. A farmer had 14 cows, 8 calves, 9 young cattle, 5 horses, a colt, a filly, 37 sheep, 14 lambs, and 19 swine. What was the total number of his live-stock?

35. A farmer bought three farms with the standing crops and the live-stock on them. For the first farm he paid $4,793 for the land, $479 for the crop, and $698 for the live-stock; for the second he paid $5986 for the land, $973 for the crop, and $546 for the live-stock; for the third he paid $8678 for the land, $1094 for the crop, and $783 for the live-stock. What was the total amount he paid for the land, for the crops, and for the live-stock, respectively? What did the whole cost him?

36. At an inspection of the Queen's Own Rifles in Toronto there were present 2 officers and 77 non-commissioned officers and men in No. 1 Company; 2 O. and 59 N-C. O. and M. in No. 2 Company; 3 O. and 60 N-C.O. and M. in No. 3 Company; 3 O. and 72 N-C. O. and M. in No. 4 Company; 3 O. and 64 N-C. O. and M. in No. 5 Company; 3 O. and 67 N-C.O. and M. in No. 6 Company; 3 O. and 60 N-C. O. and M. in No. 7 Company; 3 O. and 57 N-C. O. and M. in No. 8 Company; 3 O. and 49 N-C. O. and M. in No. 9 Company; 3 O. and 60 N-C. O. and M. in No. 10 Company; 7 officers of the Staff, and 37 musicians in the Band. What was the total strength of the battalion present at inspection?

37. A man walked 29 miles on Monday, 37 on Tuesday, 28 on Wednesday, and 19 on Thursday. How many miles did he walk altogether?

38. A man travelled 79 miles by stage, 47 miles by water, 198 miles by rail and then 67 miles on horse back. How far did he travel?

39. James hoed 29 rows of potatoes, William hoed 27 rows, Edward hoed 25 rows, and Henry hoed 47 rows. How many rows altogether did they hoe?

40. Annie picked 7 quarts of berries, Jennie p iced 9 quarts, Bertha picked 5 quarts, Mary picked 8 quarts, Harriet picked 13 quarts, and Bella picked 14 quarts. How many quarts did they pick altogether?

41. A miller bought 897 bushels of wheat and sold 136 barrels of flour in September; in October he bought 655 bushels and sold 97 barrels; in November he bought 768 bushels and sold 88 barrels; in December he bought 596 bushels and sold 194 barrels. How many bushels of wheat did he buy in the four months, and how many barrels of flour did he sell?

42. James gave 8 apples to Henry, 7 to John, 9 to Thomas, 6 to Daniel, and had 17 left. How many had he at first?

43. A merchant mixed 5 pounds of tea worth $2.15 with 8 pounds worth $4.16. What was the weight and what the value of the mixture?

44. A woman sold 19 pounds of butter for $4.75, 23 pounds for $5.30, and 25 pounds for $5.85. How many pounds did she sell, and how much did she get for the whole?

45. In a certain orchard there were 97 apple trees, 47 plum trees, 39 cherry trees, 56 peach trees, and 28 pear trees. How many trees were there in all?

46. In a certain plantation there are 498 maple trees, 1367 oak, 495 beech, 1875 elm, 196 ash, and 98 basswood trees. How many trees are there altogether?

47. A farmer has 795 sheep in one flock, 639 in a second, 478 in a third, and 639 in a fourth, and he buys 766. How many has he then in all?

48. In January 1,790,846 persons travelled by rail; in February, 984,068; in March, 2,683,705; in April, 3,708,698; in May, 4,684,792; and in June, 11,677,846. How many passengers did the railways carry during the six months?

49. The books of Moses consist of 187 chapters, the other historical books of 226, the Prophecies of 273, Job of 42, the writings of David and Solomon of 201, the four Gospels of 89, and the other books of the New Testament of 171 chapters. How many chapters are there in the whole Bible?

50. A certain book consists of four volumes. The first volume contains xxxvii+498 pages; the second, xlix+795 pages; the third, cix+688 pages; and the fourth, xciv+873 pages. How many pages in the whole book?

NOTE.—The numbers in Roman notation denote the number of pages in the preface and the contents; the numbers in Arabic notation denote the number of pages in the main body of the work.

51. In 1881 there were in Ontario 139,031 persons who had been born in England or in Wales, 130,094 in Ireland, 82,173 in Scotland, 686 in Prince Edward's Island, 3,706 in Nova Scotia, 2,801 in New Brunswick, 50,407 in Quebec, 1,435,647 in Ontario, 62 in Manitoba, 42 in British Columbia, 158 in the Canadian Territories, 45,454 in the United States, 23,270 in Germany, 4,624 in other countries, 256 at sea, and 2,211 whose birthplaces were not reported. What was the total population of Ontario in 1881?

52. How many strokes does a clock strike in telling the hours from half-past eight to half-past four?

53. How many strokes does a clock that tells the hours strike in a whole day?

54. James has 9 marbles, William has 7 more than James, John has 3 more than James and William together, and Henry has as many as all the other three boys. How many have they altogether?

55. Jane sold 7 tickets for the Sunday-school picnic, Annie sold 9, Fannie 14, Mabel 19, Jessie 4 more than Jane and Fannie together, one of the teachers sold 39, another teacher 9 more than the first one, and the superintendent 2 more than both of the teachers. How many did they sell altogether?

56. Willie had two sticks; one he threw 24 yards up the road and the other he threw 37 yards down the road. His dog Carlo was standing beside him when he threw the sticks; how far had Carlo to run to bring the sticks back to Willie, who meanwhile stood still?

57. Wingham is 74 miles north of London, which is just 27 miles less than the distance from Whitby to Haliburton. London is 121 miles west of Toronto and Whitby is 30 miles east of Toronto. How many miles is it from Wingham to Haliburton by way of London, Toronto and Whitby?

II. SUBTRACTION.

Subtraction *is the operation by which we find the number that remains when one of two given numbers is taken from the other as a part from the whole.*

The number which is taken away or subtracted is called the **Subtrahend.**

The number from which the subtrahend is taken or subtracted is called the **Minuend.**

The number resulting from the subtraction is called the **Remainder** *and also the* **Difference.**

The sign of subtraction is $-$, read *minus.* This sign, $-$, written before a number, denotes that the number is a subtrahend. Thus $4-3$ is read "four minus three," and denotes that 3 is to be subtracted from 4. Again, $63-22-13$ is read "sixty-three minus twenty-two minus thirteen," and denotes that 22 is to be subtracted from 63 and then 13 subtracted from their remainder. $47+8-12$ is read "forty-seven plus eight minus twelve," and denotes that 8 is to be added to 47 and then 12 subtracted from their sum. $29-6+9$ is read "twenty-nine minus six plus nine," and denotes that 6 is to be subtracted from 29 and then 9 added to their remainder.

Since the subtrahend and the remainder are the parts of the minuend as a whole, *the sum of the subtrahend and remainder is equal to the minuend.* Hence—

To prove the correctness of an answer in subtraction, add the subtrahend to the number found as remainder; the sum should be equal to the minuend.*

The minuend, subtrahend and remainder must all be like numbers.

---

* That is, *to test, to try, to put to proof.*

EXERCISE II.

1. Jane bought a Third Reader for 36 cents. She gave the merchant a fifty-cent piece. How much change should she get back?

2. There are 340 pages in the Third Reader and 229 in the Second. How many more pages are there in the Third than in the Second Reader?

3. Of 57 pupils present at a school examination 29 were boys. How many were girls?

4. Out of a class of 43 boys 26 were promoted. How many were left?

5. In a game of ball one side made 37 runs and the other made 29. By how many runs did the first side win?

6. In a game of cricket both sides together made 235 runs; of these the winning side made 137. How many did the other side make?

7. Seventeen swallows were sitting on a telegraph wire. A number flew away and there were 9 left. How many flew away?

8. James had 73 marbles. He lost 19 of them. How many had he then?

9. Maggie and Emma tried which could find a sunflower with the greatest number of seeds in it. Maggie found one with 279 seeds, but Emma found one with 293 seeds. How many more seeds were there in Emma's than in Maggie's?

10. There are 23,145 verses in the Old Testament and 7957 in the New Testament. How many verses are there in the whole Bible, and how many more in the Old Testament than in the New?

11. The last chapter of Isaiah is numbered LXVI.; the last Psalm CL. How many more Psalms are there than there are chapters in Isaiah?

12. John has read all the Psalms to the end of the XCVII. How many has he still to read to finish the whole hundred and fifty of them?

13. Jane has read 157 pages of her book, which consists of 324 pages altogether. How many has she still to read to finish the book?

14. Harry bought a Third Reader for 36 cents and an arithmetic for 25 cents. He gave a dollar bill to pay for them. How much change should he get back?

15. Annie bought a Third Reader for 36 cents, a geography for 65 cents, and a slate for 27 cents. She gave a two-dollar bill to pay for them. How much change should she get back?

16. Out of a bag containing 225 marbles John took two handfuls, and there were 199 marbles left. How many did John take out?

17. A man borrowed $2790 and promised to pay $285 for the loan. He repaid $764 at one time, $847 at another, and $793 at another. How much did he still owe?

18. A woman had to pay 63 cents for a pound of tea, 38 cents for 2 pounds of butter, 35 cents for a pound of coffee, 27 cents for 3 pounds of sugar, and 39 cents for 3 dozen eggs. She gave a five-dollar bill in payment. How much change should she get back?

19. Willie was first on the honor roll for April, so his father gave him 25 cents, and his mother gave him 12 cents. He spent 8 cents for a ball, 5 cents for a top, 2 cents for marbles, and 1 cent for candy. How many cents had he left?

20. During the first week of school Willie received 35 merit marks, during the second week 27, during the third week 34, and during the fourth week 29. How many did he receive in all during the four weeks, and how many did he receive less than John, who got 151 merit marks during the same four weeks?

21. Harry got 73 merit marks in March. This was 8 more than Willie got. How many did Willie get?

22. James got 473 merit marks during the term; Willie got 18 more than James, and Harry got 27 less than Willie. How many did Harry get?

23. Willie attended school 15 days in January, 17 in February, 16 in March, 16 in April, 21 in May, and 18 in June. If there were 125 school days in the six months, how many days was Willie absent from school?

24. On Monday I started on a journey of 4000 miles, and made 457 miles that day, 468 miles on Tuesday, 528 miles on Wednesday, 509 miles on Thursday, 514 miles on Friday, and 579 on Saturday. How many miles of my journey remained for me at the close of each day? How many miles had I travelled at the close of each day?

25. A drover bought 123 sheep for $710 and sold 69 of them for $475. How many had he left, and for how much must he sell them in order to get back his $710 exactly?

26. A cattle dealer bought 235 head of cattle for $5784. He sold 148 of them for $4375. How many had he left, and for how much must he sell them in order to gain $1250 on the whole transaction?

27. A butcher bought 47 oxen for $1630 and 107 sheep for $725. He sold 29 of the oxen for $1245, and the remainder for $850. He sold 48 of the sheep for $364, then 17 more of them for $150, and the remainder for $425. How much did he gain altogether?

28. A farmer had 127 acres of land, and he bought 87 acres. He afterwards sold 68 acres. How many had he left?

29. Jennie had 23 chickens more than Edith, but only 9 more than Mary. How many had Mary more than Edith?

30. Annie has one hen with 7 chickens, a second with 11 chickens, and a third with 6 chickens. Jane has two hens with 9 chickens each and a third with only 1 chicken. How many more chickens have Annie's hens than Jane's?

31. Bella had 173 nuts. She gave 19 to her sister, 17 to each of her two brothers, 23 to her father, 25 to her mother, and 22 to her cousin Ella. How many did she give away? How many had she left for herself?

32. Annie had 51 nuts. She gave 14 to Bessie, 12 to Fannie, ate 11 herself, and gave the rest to her little brother Harry. How many did she give to Harry?

33. In the first car of a railway train there were, on starting, 29 passengers; in the second, 27; and in the third, 15. At the first stopping place 19 passengers got out and 7 others got in. How many passengers were then on the train?

34. In the first car of an excursion train from London through Hamilton to Toronto there were 27 passengers; in the second car, 33; in the third, 31; in the fourth, 25; in the fifth, 32; in the sixth, 34; and in the seventh, 26. At Hamilton 8 passengers got out of each of the seven cars, and 4 got into the fourth car and 3 into the seventh. How many passengers were there then on the several cars, and how many on the whole seven?

35. A man had to put 73 head of cattle into four cars. He put 18 into the first, and 19 each into the second and third cars. How many were left to go into the fourth?

36. A man bought a horse for $97 and another for $85. He sold the two together for $163. How much did he lose on them?

37. One farmer had 157 bushels of wheat worth 172 dollars 256 bushels of oats worth 102 dollars, and 163 bushels of barley worth 106 dollars. Another farmer had 97 bushels of wheat worth 107 dollars, 311 bushels of oats worth 118 dollars, and 244 bushels of barley worth 146 dollars. Which farmer had the greater number of bushels of grain, and how many bushels had he more than the other? Which farmer's grain was worth most, and how much was his worth more than the other's?

38. Sold goods for $1225, gaining thereby $248. How much did the goods cost?

39. A man had property worth $123,273. Of this amount $15,274 was in real estate, $27,310 was in bank stock, $10,850 was in railway bonds, and the remainder was invested in mortgages. How much was invested in mortgages?

40. One day a speculator gained $7321, but next day he lost $4732. Next week he gained $5736, but immediately after lost $3143. How much more did he gain than lose in the fortnight?

41. One week a wheat buyer gained $2741, the next week he lost $713, the next week he lost $1284, but the next week he gained $925. How much more did he gain than lose during the month?

42. John has 7 marbles more than James, but 6 less than William, who has 15. Edward has as many as John and James together. How many has Edward?

43. Henry has 14 cents less than Albert, but 9 more than Thomas, who has 18 cents. How many has Fred, who has 9 less than Albert?

44. I bought four houses for $15,000. For the first I gave $725 more than for the second, but $540 less than for the third, for which I paid $4200. How much did the fourth cost me?

45. A man bought four horses for $728. For the first he paid $129, for the second $63 more than for the first, and for the third $27 less than for the second. How much did he pay for the fourth?

46. On a farm of 112 acres there were 68 acres of improved land. How many acres remained unimproved?

47. On a farm of 92 acres 26 acres were under crop, 11 acres were in pasture, and 2 acres were taken up by the garden, house and orchard; the rest was unimproved. How many acres were unimproved?

48. A man's salary is $1420 a year, and he has a property that brings him in $225 a year. If his expenses are $975 a year how much can he save?

49. A man bought 100 acres of land for $5750. He paid $1225 in cash, and gave a mortgage for the balance. For how much was the mortgage?

50. A man bought a house and a garden for $4760. He paid $1950 in cash, gave a note of hand for $825, and a mortgage for the balance. For how much was the mortgage?

51. Jones owed Smith $163; in payment he gave a horse and $49 in cash. How much was the horse reckoned at?

52. Willie had 33 marbles; he won 9 from John and lost 7 to Henry. How many had he then?

53. Jane had 35 plums, Annie had 28, and Susan had 22. Jane gave Susan enough to make her number up to Annie's. How many had Jane left for herself?

54. Harry had 25 pigeons and Bessie had 14 hens. Harry gave Bessie 9 of his pigeons in exchange for 5 of her hens. How many birds of each kind had each of them then?

55. Charlie had 29 marbles and Willie had 22. Charlie won 6 from Willie. How many had Charlie at first more than Willie, and how many more than Willie had he after winning the six?

56. Harry had 17 marbles and John had 13. Harry bought 14 and then played with John, who won 9 from him. Which of them had more than the other then, and how many more had he?

57. Albert had 57 marbles and Dick had 29. How many had Albert more than Dick? Dick won 15 marbles from Albert. Which had more than the other then, and how many more than the other had he?

58. Herbert had 52 marbles, Willie had 43, and Robert had 37. How many marbles had Herbert more than Willie, and how many more than Robert? Herbert won 4 marbles from Willie, but lost 6 to Robert. How many had each then? Willie next played with Robert and won 9 from him. How many had each now, and how many had Herbert more than each of the others?

59. James had 21 marbles more than Harry had, but Harry won 9 from him. How many more than Harry had he then?

60. James had 13 marbles more than Harry had. Harry won 4 marbles from him, and John won 3 from him also. How many more than Harry had he now?

61. Willie had 23 marbles more than Edward. Willie played with Edward and lost 7 marbles to him; Edward then played with Harry, who won 4 from him. How many then had Willie more than Edward?

62. Tom had 27 marbles more than Fred and 31 more than Dick. Tom lost 5 to Fred and 8 to Dick, and Fred lost 2 to Dick. How many then had Tom more than Fred, and how many more than Dick? How many had Fred at first more than Dick, but after the play how many had Dick more than Fred?

63. Cartier discovered Canada in July, 1534. How many years was it from then till the formation of the Dominion of Canada in July, 1867?

How many days are there:—

64. From the 12th to the 31st of May?

65. From the 1st to the 20th of June?

66. From 31st May to 20th June?

67. From 17th March to 7th May?

68. From 21st April to 2nd Dec.?

69. From 10th Oct. to 1st Feb. next following?

70. Sir Isaac Newton was born 25th Dec., 1642, and he died 20th March, 1727. How old was he when he died?

III. MULTIPLICATION.

[The pupil should be led by means of simple introductory exercises to apprehend the difference between numbers with definite group-units, such as 2 pairs of pigeons, 4 clusters of 3 cherries each, and numbers with indefinite group-units, such as 2 flocks of pigeons, 4 clusters of cherries, and to perceive that the former quantities can be expressed as numbers with the unit of the group—in the above examples, 1 pigeon and 1 cherry respectively—while the latter quantities cannot be so expressed.]

In counting any collection of objects, the unit or standard by which we count may be a single object of the kind counted, or may be a given number of such objects. Thus we may count a group of objects by one at a time, or by two at a time, or by three at a time, or by four at a time, or by any other number at a time, and say there are so many ones, or twos or threes, or fours, or whatever the number may be that we used in counting. For example, eggs are generally counted by twelves, and the number of twelves, or dozens, as they are called, is stated, not the number of single eggs. Stockings are counted by twos, and the number of pairs (twos) stated, not the number of single stockings. But if the number of twos, or threes, or fours, or whatever the number in each *count* may be—that is, the number of *units*—be stated, it may be required to find how many single objects there are. The number of these is found by the operation called multiplication.

Multiplication *enables us to find conveniently the sum of a stated number of repetitions of any given number.*

The number to be repeated (or multiplied) *is called the* **Multiplicand.**

The number which states how often the multiplicand is to be repeated (how many of it is to be taken) *is called the* **Multiplier.**

The number resulting from the multiplication is called the **Product.**

The multiplicand and multiplier, taken together, are called the **Factors** *of the product.*

The sign of multiplication is $\times$, read "multiplied by." This sign $\times$ written before any number denotes that the number is a multiplier. Thus 3×2 is read "three multiplied by two," or

"twice three," and denotes two *threes*, that is, the sum of two threes ; 5×4 is read "five multiplied by four," or "four times five," and denotes four *fives*, that is, the sum of four fives ; 7 pencils $\times 5$ is read "seven pencils multiplied by five," or "five times seven pencils," and denotes 5 bundles *of seven pencils each;* 12 bricks $\times 4$ is read "twelve bricks multiplied by four," or "four times twelve bricks," and denotes 4 heaps *of twelve bricks each.*

Since a unit may be either abstract or concrete, the **multiplicand**, which is the unit of the multiplier, **may be either abstract or concrete.**

Since the multiplier has the multiplicand for unit, **the multiplier taken without the multiplicand must be abstract.**

The product and the multiplicand must be like numbers.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

[Pupils should so learn the table as to be able to repeat it both by columns and by rows; *e.g.*, twice 1 is 2, twice 2 is 4, twice 3 is 6, twice 4 is 8, etc., and once 2 is 2, twice 2 is 4, thrice 2 is 6, four times 2 is eight, etc. They may thus be led to perceive the truth of the important principle:—

The product of two abstract factors is the same, whichever factor is taken as multiplier.]

EXERCISE III.

1. Willie was at school 6 hours each day for 22 days in March. How many hours was he at school that month?

2. There are 60 minutes in an hour. How many minutes are there in six hours?

3. How many minutes are there in a day?

4. How many minutes are there in a week?

5. There are 24 hours in a day. How many hours are there in July?

6. How many hours are there in a year—365 days?

7. Harry attended school on 17 days in January, and had to walk 3 miles each day to do so. How many miles did he walk to attend school that January?

8. Annie walked a mile to school every school-day and a mile back again. How many miles did she thus walk in a week of 5 school-days?

9. A railway train ran for 4 hours at the rate of 27 miles an hour. What distance did it run?

10. George takes 2350 steps to the mile. How many steps will he take in walking 3 miles?

11. Fred takes 2460 steps to the mile. How many steps will he take in walking 3 miles a day for 5 days?

12. A cat has 18 toes. How many toes will 18 cats have?

13. There are eight boys in Willie's class, including Willie himself. Each boy has twenty-eight teeth. How many teeth have they altogether?

14. A spider has 8 legs and a fly has 6. How many legs will 6 spiders and 8 flies have?

15. How many feet altogether have 3 horses, 4 cows, and 5 sheep?

16. A mail-carrier drove daily from *A* to *B*, 4 miles; from *B* to *C*, 3 miles; from *C* to *D*, 5 miles; and from *D* back to *A*, 5 miles. How many miles did he drive every week omitting Sundays?

17. James walked 8 miles a day on 25 days in January, 23 in February, and 26 in March. How many miles in all did he walk during the 3 months?

18. An acre of land contains 4840 square yards. How many square yards are there in 37 acres?

19. Find the cost of 27 tons of iron at $39 the ton?

20. At 27 bushels of wheat to an acre, how many bushels would there be on 36 acres?

21. At 23 bushels to an acre, how many bushels would there be to the square mile of 640 acres, deducting 43 acres for roads, fences, and waste land?

22. How many bushels of wheat could be raised in a township containing 78 square miles of 640 acres each, allowing 47 acres in each square mile for roads, fences, and waste land, the wheat averaging 23 bushels to an acre?

23. A drover bought 37 head of cattle at $48 each. How much did he pay for them?

24. A woman bought 6 pounds of tea at 67 cents a pound, 18 pounds of sugar at 12 cents a pound, 2 pounds of coffee at 33 cents a pound, 8 pounds of cheese at 14 cents a pound, 13 pounds of butter at 23 cents a pound, and 9 dozen eggs at 19 cents a dozen. Find the price of the whole.

25. How much money would be required to pay $500 each to 798 men?

26. Find the strength of an army consisting of 97 regiments of 873 men each.

27. How many bricks will there be in 97 feet of wall if each foot require 78 bricks?

28. What will be the total issue of a newspaper in 13 weeks of 6 days each, if the daily issue be 23,785 copies?

29. A merchant bought 768 pounds of cheese at 7 cents a pound, 287 pounds of butter at 19 cents a pound, and 178 dozen eggs at 13 cents a dozen. Find the cost of the whole.

30. A merchant bought 7 chests of tea, each weighing 68 pounds, at 37 cents a pound; 3 barrels of sugar, each weighing 226 pounds, at 9 cents a pound; 17 boxes of raisins, each weighing 38 pounds, at 13 cents a pound; and 44 canisters of spices, each weighing 24 pounds, at 17 cents a pound. Find the cost of the whole.

31. A man has a chest of tea which at first contained 87 pounds, but 29 pounds have been taken out of it. How much is the remaining tea worth at 63 cents the pound?

32. A man bought two farms, one containing 167 acres at $73 an acre, the other containing 79 acres at $87 an acre. How much did the two cost him?

33. James bought 4 oranges at 4 cents each. What change should he get back out of a twenty-five-cent piece which he gave in payment?

34. A man bought 9 cords of wood at $4 a cord, and gave 4 ten-dollar bills in payment. How much change should he receive?

35. What is the weight of a train consisting of 17 cars, each weighing 22,375 pounds, and an engine and tender weighing together 147,800 pounds?

36. James knows of a butternut tree with 34 bunches on it with 5 nuts to a bunch, 47 bunches with 4 nuts to a bunch, 18 bunches with 3 nuts to a bunch, and 7 bunches with 2 nuts to a bunch. How many bunches were there on the tree, and how many butternuts were there?

37. If out of a salary of $1300 a year a man pay $180 for board, $197 for clothing, $167 for books, and $238 for other expenses, how much can he save in seven years?

38. Charles had saved the sum of 29 cents. His father then gave him five times as much. How much had he then?

39. Fannie had saved 19 cents. She was first on the Honor-Roll for May, so her father gave her five times and her mother twice as much as she had saved. How much had she then?

40. Bertha picked 47 plums and her brother Thomas picked 9 more than five times as many. How many did Thomas, and how many did both pick?

41. Annie bought a book for 17 cents, and a box of paints for four times as much. How much did both cost her?

42. Emma bought a doll for 25 cents and a doll's carriage for five times as much. How much did both cost her?

43. The furniture in a house was worth $1837, the house itself was worth twice as much, and a library in the house was worth twice as much as the house. How much was the whole worth?

44. Jane's hen has 13 chickens; Annie's 5 hens have four times as many all but 4. How many chickens have Annie's hens?

45. If a certain farm-house be worth $720, and the farm and barns be worth $400 less than five times as much, and the stock and standing crops be worth $125 more than thrice as much as the house, how much will the whole be worth?

46. A man had four rolls of five-dollar bills. In the first roll there were 17 bills; in the second, 25 bills; in the third, 24 bills; and in the fourth, 33 bills. What was the total value of the four rolls?

47. A man had three rolls of five-cent pieces. In the first there were 60 pieces; in the second, 75 pieces; and in the third, 118 pieces. What was the value of the whole?

48. In a certain book there are 239 pages of 37 lines each, averaging 46 letters to a line. How many lines and how many letters are there in the book?

49. In a certain school-room there were 7 rows of desks with 6 desks to the row, each desk accommodating 2 pupils. How many desks were there in the room, and how many pupils would they accommodate?

50. In a certain school-house there are 29 windows; in each window there are 4 rows of panes with 3 panes in each row. How many panes are there in each window, and how many in the whole?

51. In a field of Indian corn there were 67 rows with 78 hills in every row. How many hills were there? If these a eraged 7 ears to a hill, how many ears did the field yield?

52. In a certain house of 4 stories there are in each storey 15 windows in the front, 8 windows at each end, and 14 windows in the rear. In each window there are 12 panes of glass. How many panes are there in each storey and how many in the whole, and what is their value at 17 cents each?

53. A drover bought 73 sheep at $6 each and sold the whole for $575. How much did he gain thereby?

54. A man bought 27 horses at $137 each and sold them at $176 each. How much did he gain on each and how much on the whole?

55. A drover bought 89 head of cattle at $39 the head and sold them at $48 the head. What was his gain on the whole?

56. A drover bought 748 head of cattle at $43 each and sold them at $61 each, having meanwhile been at an expense of $816 on account of the cattle. What was his net gain?

57. A merchant bought 78 yards of cloth at 63 cents a yard and sold 14 yards to one man, 9 yards to a second man, 15 yards to a third man, 13 yards to a fourth man, and 18 yards to a fifth man, all at 95 cents a yard; the remainder he sold at 87 cents a yard. How much did he gain on the whole?

58. A man bought a span of horses for $275; he hired them out at $12 per week, but paid $5 per week for their keep. At the end of 7 weeks he sold them for $250. Did he gain or did he lose on them? How much?

59. James is 9 years old, and his father is four times as old all but a year. How old is his father?

60. Annie is 12 years of age; her mother is two years more than thrice as old. How old is her mother?

61. How many days' work will 36 men do in 27 days?

62. 873 men built a section of railway in 249 days. How many days' labor did the building of the section require?

63. If a gang of 9 men can dig 37 yards of a trench in 6 days, how many yards could 8 such gangs dig in the same time? How many days' labor would the 8 gangs do?

64. If 16 men put up 13 miles of wire-fencing in a week, how many miles could thrice as many men put up in 4 weeks?

IV. DIVISION.

[Introductory exercises should be given in both kinds of Division, but they should not be mingled indiscriminately. In reading the results the divisor and the quotient should always be read as co-factors. Thus the result of the division 3)15 cents should be read 3 times 5 cents is 15 cents; the result of the division 3 plums)15 plums should be read 5 times 3 plums is 15 plums; and the result of 3)15, in which both divisor and dividend are abstract, should be read both as 3 fives and as 5 threes.]

Division *is the operation by which we find the number which, taken as co-factor with one of two given numbers, would yield the other given number as product.*

The number found by the division is called the **Quotient.**

That one of the given numbers which is co-factor of the quotient is called the **Divisor.**

That one of the given numbers which is equal to the product of the divisor and the quotient is called the **Dividend.**

Since in multiplication we generally have so many THINGS repeated so many TIMES there will be *two* kinds of division, according as the number of *things* or the number of *times* is given as divisor.

In the first kind of division we find the number which, taken a given number of times, would make up a given number. In such case *the divisor tells how many times* the quotient must be taken to make up the dividend. The *divisor* must therefore be abstract, and the dividend and the quotient must be like numbers.

In the second kind of division we find how many numbers, each equal to a given number, would, either by themselves or else along with a number (called the remainder) to be found, and less than the given number, make another given number. In this case *the quotient tells how many times* the divisor must be taken in order that the product increased by the remainder, if there be any, may be equal to the dividend. The *quotient* must therefore be abstract, and the dividend, the divisor, and the remainder, if there be any, must be like numbers.

The sign of division is ÷, read "divided by." This sign ÷, written before any number, denotes that the number is a divisor. Thus 12 apples ÷ 2 is read "twelve apples divided by two," and denotes one of the parts resulting from dividing a collection of 12 apples into 2 equal parts. 15 cents ÷ 3 is read "fifteen cents divided by three," and denotes one of the sums resulting from dividing the sum of 15 cents into 3 equal sums. 20 penholders ÷ 4 penholders is read "twenty penholders divided by four penholders," and denotes the *number* of the bundles of 4 penholders each that could be made out of a bundle of 20 penholders. 30 boys ÷ 5 boys is read "thirty boys divided by five boys," and denotes the *number* of the groups of 5 boys each that could be made out of a group of 30 boys.

To prove the correctness of an answer in division, multiply the divisor and quotient together, and to the product add the remainder, if there be any; the result should be equal to the dividend.

EXERCISE IV.

1. How many oranges at 4 cents each can John buy for 56 cents?

2. James bought 12 oranges for 36 cents. How much did they cost him apiece?

3. A gentleman gave 91 cents to be divided equally among 7 boys. How much should each boy receive?

4. Herbert was given 24 cents to be divided equally among himself and his three brothers. How many cents should each get?

5. How many lengths of 6 yards each can be cut from a piece of silk 42 yards long?

6. If 96 pounds of flour be made up into 32 loaves, what weight of flour will that allow to each loaf?

7. How many four-dollar bills will amount to $144?

8. A butcher bought 19 lambs for $57. How much apiece did they cost him?

9. A farmer got 203 bushels of wheat off a seven-acre field. How many bushels was that to an acre?

10. A farmer sold his farm of 137 acres for $8631. How much was that an acre?

11. How many times is $17 contained in $11,917?

12. What sum taken 17 times will amount to $11,917?

13. If $34,051 be divided into 17 equal parts, what will be the amount of one of them?

14. A box contained 1128 eggs. How many dozen eggs were there in the box?

15. A grocer bought 1416 eggs at 18 cents a dozen. How much did they cost him?

16. A mile contains 63,360 inches. How many steps of 20 inches each will George have to make to walk a mile?

17. Thomas walked a mile and a half (equal to 95,040 inches) in 3520 steps. How many inches did he take each step?

18. If 7 dozen eggs cost 168 cents, what was the price per dozen? How much is that per egg?

19. The wages of 13 men for one week were $97.50. How much did a man earn per day?

20. The expense of building a bridge was $8743, and of opening a road was $2163. The total expense was borne equally by seven townships. What was the share of each?

21. A man receives a salary of $1200 a year. Out of this he saves $212 each year. How much does he spend per week, counting 52 weeks to the year?

22. A contractor requires one million bricks. He has 559,941 already. How many loads of 437 bricks each does he need to make up the full number?

23. How many bags of flour, each containing 25 pounds, can be made out of 75 barrels of flour, each containing 196 pounds?

24. A bushel of wheat weighs 60 pounds and a bushel of oats 34 pounds. How many bushels of oats will weigh as much as 187 bushels of wheat?

25. How many 11-foot panels in a mile (5280 feet) of fencing?

26. How many boards, each 12 feet long, will be required to build 1320 feet of fencing 5 boards high?

27. Seven men have an equal interest in a farm of 107 acres. They sell it at $56 an acre. How much should each receive?

28. In a school-room there were 72 seats arranged in 6 rows. How many seats were there in each row?

29. In a school-room there were 6 rows of seats, and 57 boys filled all the seats but 3. How many seats were there in each row?

30. Fred had 75 cents. He bought 2 dozen oranges and had 3 cents left. How much were the oranges apiece?

31. Willie has 69 nuts. He gives 12 to each of his brothers and keeps the smallest share for himself. How many brothers had he? How many did he keep for himself?

32. A lady sent a bag of apples to be divided among the pupils of a class consisting of ten boys and a girl. It was decided to share them equally as far as possible without cutting any of the apples, and if any then remained to give them to the girl. There were 180 apples in the bag. How many did each boy receive, and how many did the girl receive?

33. An excursion boat can cary 125 passengers per trip. How many trips must it make to carry 1357 passengers? If it carried the full number every trip but the last one, how many did it carry the last trip?

34. A steamer which is not allowed to take more than 125 passengers per trip has to carry 2337 persons. How many trips at least must it make to do so? If it carry the same number on each of these trips, how many will that be?

35. How much would 20 apples cost at 5 for 2 cents?

36. How much would 12 peaches cost at 3 for 5 cents?

37. How much would a dozen pears cost at 2 for 5 cents?

38. If 35 marbles be divided equally among 5 boys, how many will two of the boys get?

39. If $116,323 be divided into 89 equal parts, what will be the amount of 49 of them?

40. James has 72 marbles, John has half as many, and Willie one-third as many as John. How many has Willie?

41. Thomas has 66 cents, James has one more than half as many as Thomas, Harry one more than half as many as James, Willie one more than half as many as Harry, Bertha one more than half as many as Willie, and Jessie one more than half as many as Bertha. How many has Jessie?

42. How long would a man take to walk 18 miles at the rate of 3 miles an hour?

43. How long would sound take to travel 9056 feet at the rate of 1132 feet per second?

44. How long would it take a train, running at an average of 32 miles an hour, to go from Buffalo to Goderich, a distance of 160 miles?

45. How many days would a man take to walk 156 miles at the rate of 3 miles an hour for 8 hours a day?

46. A man travelled 196 miles. For a quarter of the way he rode at the rate of 7 miles an hour, and for the rest of the way he walked at the rate of 3 miles an hour? How many days did he take to make the journey if he travelled but 8 hours each day?

CHAPTER III.

COMPOUND NOTATION SYSTEMS.

I. TABLES OF MEASURES.

MONEY, OR MEASURES OF VALUE.

100 cents (ct.)=1 dollar . . . . . ($)

MEASURES OF WEIGHT.

Avoirdupois Weight is used for all the ordinary purposes of weighing.

16 ounces (oz.)=1 pound . . . . . (lb.)
2000 pounds =1 ton . . . . . (T.)

100 pounds is called a cental or hundredweight, denoted by *cwt.*

In weighing very small quantities a weight called a *grain* is used.

7000 grains (gr.) =1 pound.

LINEAR MEASURE.

Linear Measure, or **Long Measure**, is used in measuring length (width, thickness) and distance.

12 inches (in.)=1 foot . . . . . (ft.)
3 feet =1 yard . . . . . (yd.)
1760 yards =1 mile . . . . . (mi.)

A *hand*, used in measuring the height of horses, is equal to 4 inches.

A *fathom* is equal to 6 feet.

A rod, pole or perch is equal to $5\frac{1}{2}$ yards. Distances less than a mile are sometimes stated in rods.

SURFACE MEASURE.

Surface Measure or Square Measure is used in measuring surfaces or areas, as of land, painting, plastering, flooring.

144 square inches (sq. in.)	= 1 square foot	(sq. ft.)
9 square feet	= 1 square yard . . .	(sq. yd.)
4840 square yards	= 1 acre	(A.)
640 acres	= 1 square mile . . .	(sq. mi.)

A square rod, pole or perch is equal to $30\frac{1}{4}$ square yards.

In measuring land, surveyors use **a chain 22 yards long,** divided into **100** equal parts called **links.**

10,000 square links (sq. l.)	= 1 square chain . . .	(sq. ch.)
10 square chains	= 1 acre	(A.)

CUBIC MEASURE.

Cubic Measure, or Solid Measure, is used in measuring volumes and capacities, as the volume or solid contents of timber, stone, earthwork, and boxes of goods, and the capacity of boxes, bins, rooms, etc.

1728 cubic inches (cu. in.)	= 1 cubic foot	(cu. ft.)
27 cubic feet	= 1 cubic yard	(cu. yd.)

Firewood and rough stone are measured by the **cord of 128 cubic feet,** which is equal to a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

MEASURE OF CAPACITY.

Measure of Capacity is used in measuring milk, oil, molasses, water, and other liquids, and such articles as grain, fruit, roots, salt, and lime.

2 pints (pt.)	= 1 quart	(qt.)
4 quarts	= 1 gallon	(gal.)
2 gallons	= 1 peck	(pk.)
4 pecks	= 1 bushel	(bu.)

The capacity of cisterns, reservoirs and the like is often expressed in barrels (bbl.) of $31\frac{1}{2}$ gallons each, or in hogsheads (hhd.) of 63 gallons each.

The peck and the bushel are not used in measuring *liquids*, but only in measuring *dry articles*, such as grain and fruit.

A cubic foot contains nearly 25 quarts.

A gallon of pure water weighs 10 pounds.

The legal bushel of certain substances is determined not by measure, but by weight. These weights are given in the following table: —

Blue Grass Seed	14 lb.	Indian Corn	56 lb.
Oats	34 lb.	Rye	56 lb.
Malt	36 lb.	Wheat, Beans, Peas, and Red Clover Seed	60 lb.
Castor Beans	40 lb.		
Hemp Seed	44 lb.		
Barley	48 lb.	Potatoes, Turnips, Carrots, Parsnips, Beets and Onions	60 lb.
Buckwheat	48 lb.		
Timothy Seed	48 lb.		
Flax Seed	50 lb.	Bituminous Coal	70 lb.

A barrel of flour contains 196 lb.

A barrel of pork or beef contains 200 lb.

MEASURES OF TIME.

60 seconds (sec.)	= 1 minute	(min.)
60 minutes	= 1 hour	(hr.)
24 hours	= 1 day	(da.)
7 days	= 1 week	(wk.)
365 days	= 1 common year	(yr.)
366 days	= 1 leap year.	

The leap years are those whose dates are exactly divisible by 4, except in the case of the even hundreds; these must be exactly divisible by 400. Thus 1880, 1884, 1888, 1892, 1600, and 2000 were or will be leap years; 1881, 1883, 1886, 1890, 1800, and 1900 were not or will not be leap years.

The year is divided into twelve parts called months; of these, seven consist of 31 days each, four consist of 30 days each, and one (February) consists of 28 days—in leap years of 29 days. The lengths of the several months may be remembered from the following rhymes:—

Thirty days have September,
April, June, and November,
February has twenty-eight alone;
All the rest have thirty-one;
But leap year coming once in four,
February then has one day more.

The civil day begins and ends at 12 o'clock midnight. A.M. denotes time before noon; M., at noon; P.M., after noon.

ANGULAR MEASURE.

Angular Measure is used in measuring angles, the arcs of circles, latitude and longitude, the motions of the heavenly bodies, etc.

60 seconds (″)	=1 minute	(′)
60 minutes	=1 degree	(°)
90 degrees	=1 quadrant or right angle.	
4 quadrants, or 360 degrees	=1 circle or whole circuit.	

COUNTING.

In **counting** certain classes of articles

12 articles=1 dozen	(doz.)
12 dozen =1 gross	(gro.)
20 articles=1 score	(sc.)

In **counting sheets of paper**

24 sheets=1 quire	(qr.)
20 quires=1 ream	(rm.)

A Denominate Number *is a concrete number that expresses value, weight, or measure of any kind.*

The unit or units in which a denominate number is expressed are called its *Denominations*. Thus the denomination of $5 is the dollar, that of 7 ft. is the foot, and those of 36 lb. 8 oz. are the pound and the ounce.

A unit of greater value, weight or measure than another is said to be of a higher denomination than that other. Thus the dollar is of a higher denomination than the cent, the pound than the ounce, the mile than the yard, and the day than the minute.

A number expressed in one denomination only is called a *Simple Denominate Number*. Thus 5 gal., 6 sq. in., and 23 hr. are simple denominate numbers.

A number expressed in two or more denominations is called a *Compound Denominate Number*, or, briefly, a *Compound Number*. Thus 49 lb. 4 oz., 17 yd. 2 ft. 10 in., and 3 da. 4 hr. 25 min. 18 sec. are compound numbers.

In expressing a compound number the denominations should be arranged in order from the highest to the lowest.

[It is left to the teacher to give exercises in the notation of compound numbers corresponding to the first four classes of exercises in Arabic notation, mentioned on page 7.]

EXERCISE V.

Which of the following numbers are simple and which are compound:

1. 4 lb.	*5.* 18 cords.	*9.* 3° 37′ 30″.
2. 3 pt.	*6.* 76 gal. 2 qt.	*10.* 74 bu. 1 pk. 2 qt.
3. 8 mi. 465 yd.	*7.* 45 cu. ft. 764 cu. in.	*11.* 878 cu. yd.
4. 1244 sq. yd.	*8.* 5 A. 2420 sq. yd.	*12.* 11 T. 1850 lb.

Arrange the following compound numbers so that the denominations in each shall be in regular order, beginning with the highest:

13. 6 in. 4 yd. 2 ft.	*17.* 2 sq. ft. 36 sq. in. 4 A. 531 sq. yd.
14. 4 T. 7 oz. 1623 lb.	*18.* 77 cu. in. 17 cu. yd. 7 cu. ft.
15. 426 yd. 6 in. 1 ft. 5 mi.	*19.* 1 pt. 1 gal. 2 qt. 3 pk. 5 bu.
16. 6 min. 3 hr. 2 da.	*20.* 7 sq. ch. 19 A.

II. REDUCTION.

Reduction *is the process of changing a number which expresses a quantity in terms of one or more units to a number which expresses the same quantity in terms of one or more other units.*

Reduction from units of higher to units of lower denomination is called *Reduction Descending.*

Reduction from units of lower to units of higher denomination is called *Reduction Ascending.*

REDUCTION DESCENDING.

EXERCISE VI.

1. Reduce $8 to cents.
2. Reduce $97 to cents.
3. How many cents are there in $100?
4. How many cents are there in $7001?
5. Reduce 7 feet to inches.
6. Express 4 yards in feet.
7. How many pounds are there in 77 tons?
8. How many hours are there in 28 days?
9. Express 7 acres in square yards.
10. How many pecks would make 8 bushels?
11. How many ounces would weigh as much as 12 pounds?
12. Reduce 63 cubic feet to cubic inches.
13. How many ounces do 19 tons weigh?
14. How many feet are there in 3 miles?
15. How many pints are there in 24 gallons?
16. How many minutes are there in four weeks?
17. How many sheets are there in 6 reams?
18. Reduce 124 square yards to square inches.
19. Reduce 36° to seconds of arc.
20. Express 47 bushels in quarts.
21. How many ounces of tea are there in a chest containing 147 pounds?
22. How many pints of vinegar are there in a barrel containing 36 gallons?
23. How many cubic feet of wood would make 24 cords?
24. How many ounces would 48 bushels of wheat weigh?
25. How many inches is it by railway from Toronto to Hamilton, a distance of 39 miles?

26. Reduce $8.47 to cents. *28.* Reduce $0.07 to cents.

27. Reduce $70.07 to cents. *29.* Reduce $400.10 to cents.

30. How many ounces are there in 74 lb. 8 oz.?

31. Reduce 44 T. 1650 lb. 7 oz. to ounces.

32. Reduce 31 gal. 2 qt. to pints.

33. Reduce 23 hr. 56 min. 4 sec. to seconds.

34. How many seconds are there in 365 da. 5 hr. 48 min. 49 sec.?

35. How many min. of arc are there in 43° 39′?

36. How many inches are there in 4 yd. 2 ft. 6 in.?

37. Reduce 27 sq. yd. 8 sq. ft. 96 sq. in. to square inches.

38. Reduce 23 cu. yd. 18 cu. ft. to cubic feet.

39. What will 7 gal. 2 qt. of maple syrup cost at 27 cents a quart?

40. What will 7 gross 8 doz. buttons cost at 13 cents a dozen?

41. What will 23 mi. of telegraph wire cost at 4 cents a foot?

42. What will 37 sq. m. 450 A. of land be worth at $49 an acre?

43. What will 27 hogsheads of molasses be worth at 15 cents a quart, if each hogshead contains 61 gallons?

44. How many yards of fence will it take to enclose a tract of land measuring 7 mi. 1589 yd. around?

45. What would be the value of 9 rm. 10 qr. of paper at one cent per sheet?

46. What is the value of 19 bu. 3 pk. 1 gal. of cherries at 9 cents a quart?

47. How many quart boxes will be required to hold 9 bu. 3 pk. 1 gal. 1 qt. of strawberries?

48. How many pint bottles will be required to hold 62 gal. 3 qt. 1 pt. of vinegar?

49. How many minutes are there in the month of January?

50. How many minutes were there in February, 1884?

51. How many steps a yard long each will a man need to make to walk 3 mi. 630 yd.?

52. A grocer bought a barrel of vinegar containing 36 gal. 3 qt. for $16, and sold it for 18 cents a quart. How much did he gain?

53. A fruit dealer bought 6 barrels of cranberries, each containing 2 bu. 1 pk., at $7 a barrel. He retailed them at 18c. a quart. How much did he gain?

54. What would be the cost of laying a platform to cover 1 A. 2299 sq. yd. at 50 cents a square yard?

55. What will be the cost of 7 yd. 1 ft. of lead pipe, 6 lb. to the foot, at 16 cents a pound?

56. How many tiles, each a foot long, will be required for 256 yd. of tile drain?

57. What will be the cost of digging a drain 346 yd. long at 20 cents a foot?

58. It is 24,902 mi. 214 yd. around the earth at the equator. How many steps of a yard each would a man have to make to walk that distance?

59. How many hills of Indian corn can be planted in a ten-acre field, allowing a square yard to each hill?

60. How many persons could stand on 1 A. 1826 sq. yd., allowing 3 persons to each square yard?

61. How many square feet of plank would it need to cover a play-ground whose area is 1378 sq. yd. 8 sq. ft.?

REDUCTION, ASCENDING.

EXERCISE VII.

Reduce

1. 72 in. to ft.
2. 24 pt. to gal.
3. 711 pt. to gal., etc.
4. 945 ct. to $ and ct.
5. 1602 ct. to $ and ct.
6. 830 ct. to $ and ct.
7. 7000 ct. to $.
8. 10,000 ct. to $.
9. 4010 ct. to $ and ct.
10. 678 oz. to lb. and oz.
11. 7460 lb. to T. and lb.
12. 478645 oz. to T., etc.

How many bushels are there in

13. 1000 lb. of wheat?
14. 1000 lb. of oats?
15. 1000 lb. of barley?
16. 1000 lb. of Indian corn?
17. 3750 lb. of wheat?
18. 2780 lb. of peas?
19. 1890 lb. oats?
20. 2567 lb. of rye?
21. 2745 lb. of wheat?
22. 3944 lb. of buckwheat?
23. 1476 lb. of barley?
24. 1744 lb. of oats?
25. 1968 lb. of timothy seed?
26. 2743 lb. of wheat?
27. 1679 lb. of Indian corn?
28. 7236 lb. of peas?
29. 1763 lb. of beans?
30. 3996 lb. of carrots?
31. 1843 lb. of oats?
32. 4444 lb. of wheat?

33. A cubic foot of water weighs 1000 oz. How many pounds will a cubic yard weigh?

34. A cubic foot of granite weighs 168 lb. How many tons would a solid cord of granite weigh?

35. How many tons of provisions would be required to feed 379 men for 3 years if each man be allowed 52 oz. a day?

36. How many acres will be required to raise 5000 bu. of carrots at the rate of 8 sq. yd. to a bushel?

37. A grocer paid $7.20 for a barrel of vinegar, and found that it cost him 3 cents a pint. How many gallons were there in the barrel?

38. A man sold at 20 cents a quart a barrel of molasses which cost him $23.40, and gained thereby $5.40. How many gallons were there in the barrel?

39. A brick weighs about 4 lb. What would be the total excess of weight of a million bricks if each brick exceeded the four-pound standard by one ounce?

40. In quick marching, soldiers take 120 steps of 30 in. each per minute. At that rate how far would they march in an hour?

41. At the "double" soldiers take 165 steps of 33 in. each per minute. At that rate how far would they march in 12 minutes?

42. The length of the longer diameter of the earth at the equator is 41,853,258 ft.; that of the shorter diameter is 41,850,210 ft. Express these in miles, yards, etc.

43. The length of the polar diameter of the earth is 41,708,954 ft. Express this in miles, yards, etc.

44. The length of a degree of longitude at the equator is 365,231 ft. Find the length in miles, etc., of 360 such degrees.

45. A wheel 154 in. in circumference made 7286 revolutions in rolling a certain distance. How many miles did it roll?

46. How many square yards would be occupied by a brigade of 8239 men, allowing 4 sq. ft. to each man?

47. What area of ground would require 1001 cu. yd. of gravel to cover it at the rate of 7 cu. yd. to 36 sq. yd.

48. How many acres would the total year's issue of a newspaper cover when spread out, if the size of each sheet were 722 sq. in., and there were 260 issues of 2 sheets each, and 52 issues of 4 sheets each, and 19,965 copies of each issue?

49. At what time was it exactly a million seconds past 9 a.m. on 1st March, 1893?

50. How many miles are there in 4,014,489,600 in.? How many square miles are there in 4,014,489,600 sq. in.?

51. What length of sidewalk 4 planks wide would 10,000 planks each 13 ft. long lay?

III. COMPOUND ADDITION.

Compound Addition *is the operation of finding the sum of two or more similar compound numbers.*

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EXERCISE VIII.

1. I owe $19.45 to one man, $26.58 to another, and $47.38 to a third. How much do I owe to all three?

2. A grocer sold 44 lb. 8 oz. of cheese on Monday, 38 lb. 9 oz. on Tuesday, 64 lb. 11 oz. on Wednesday, 49 lb. 4 oz. on Thursday, 36 lb. 12 oz. on Friday, and 93 lb on Saturday. What weight of cheese did he sell during the week?

3. Find the total weight of 5 car-loads of coal weighing respectively 14 T. 1763 lb., 15 T. 485 lb., 13 T. 1928 lb., 15 T. 1343 lb., and 14 T. 791 lb.

4. Three fields have an area respectively of 19 A. 4235 sq. yd., 15 A. 2223 sq. yd., and 9 A. 3861 sq. yd. What is the total area?

5. What is the entire length of a railway consisting of 5 different lines measuring respectively 167 mi. 1019 yd., 97 mi. 351 yd., 126 mi. 1537 yd., 67 mi. 1094 yd., and 48 mi. 1467 yd?

6. A farmer sold 35 bu. 2 pk. 1 gal. 1 qt., 29 bu. 3 pk. 1 gal. 3 qt., 18 bu. 1 gal., 19 bu. 3 pk., and 37 bu. 1 pk. 1 gal. 3 qt. How much did he sell in all?

7. At a certain mill 19 T. 1743 lb. coal were burned in one week, 23 T. 1645 lb. the next, 22 T. 974 lb. the next, and 18 T. 1468 lb. the next. What was the weight of the coal burned during the four weeks?

8. A merchant sold 47 gal. 3 qt. 1 pt. of coal oil and had 19 gal. 2 qt. 1 pt. left. What quantity had he at first?

9. A farmer sold 5 loads of oats containing respectively 47 bu. 18 lb., 55 bu. 19 lb., 48 bu. 27 lb., 45 bu. 25 lb., and 46 bu. 15 lb. What was the total quantity he sold?

10. Find the total quantity of wood in four piles containing respectively 17 cords 98 cu. ft., 49 cords 4 cu. ft., 25 cords 45 cu. ft., and 36 cords 112 cu. ft.

11. Find the sum in acres, etc., of 10,000 sq. ch., 10,000 sq. yd., 10,000 sq. ft., and 10,000 sq. in.

12. A farm consists of 27 A. 4466 sq. yd. of land under grain, 4 A. 2917 sq. yd. in roots, 7 A. 2087 sq. yd. under hay, 12 A. 2352 sq. yd. under pasture, 37 A. 2934 sq. yd. woodland, and 4 A. 4750 sq. yd. in orchard, garden, barnyard, and building sites. What is the total area of the farm?

13. A mistake was made in adding up an account. It was made out to amount to \$74.93, which was less than the correct amount by \$9.16. What was the correct amount?

14. A man travelled 97 mi. 1000 yd. by railway, 45 mi. 1090 yd. by steamboat, and 9 mi. 880 yd. on horseback. How far did he travel altogether?

15. A farmer harvested 469 bu. 2 pk. 1 gal. 2 qt. of wheat, 379 bu. 2 pk. of oats, 134 bu. 1 gal. 1 qt. of rye, 97 bu. 3 pk. 3 qt. of barley, and 196 bu. of Indian corn. What was his total grain crop?

16. A man walks 7219 yd., 6947 yd., 6894 yd., 6748 yd., 6536 yd., and 5977 yd. in six successive hours. How many miles, etc., did he walk in all?

17. Find the total weight of the following nine loads of wheat, and also the total number of bushels in them :—

No. 1. 27 bu. 18 lb.	No. 4. 25 bu. 54 lb.	No. 7. 24 bu. 47 lb.
No. 2. 19 bu. 44 lb.	No. 5. 26 bu. 17 lb.	No. 8. 22 bu. 36 lb.
No. 3. 25 bu. 31 lb.	No. 6. 21 bu. 35 lb.	No. 9. 29 bu. 48 lb.

18. A rectangular playground is 38 yd. 2 ft. 6 in. long and 32 yd. 1 ft. 9 in. wide. What is the total length around it?

19. A school-room is 29 ft. 3 in. long by 24 ft. 7 in. wide. Find the total length around it in yards, etc.

20. In building a house the cost was as follows :—Bricks, \$148.75; lime, \$38.50; sand, \$8.40; woodwork, \$374.98; cartage, \$94.65; wages, \$974.57; and extras and miscellaneous, \$173.48. The site cost \$325, and fencing and draining it \$49.64. What was the total cost?

21. A man travelled 38 mi. 429 yd. one day, 24 mi. 785 yd. the next day, and still had 46 mi. 376 yd. to go to finish his journey. What was the length of that journey?

22. A farm consists of 8 fields of the following areas :—

No. 1. 7 A. 3872 sq. yd.	No. 5. 16 A. 2904 sq. yd.
No. 2. 13 A. 1331 sq. yd.	No. 6. 13 A. 2057 sq. yd.
No. 3. 19 A. 1694 sq. yd.	No. 7. 9 A. 4114 sq. yd.
No. 4. 19 A. 3630 sq. yd.	No. 8. 5 A. 2662 sq. yd.

What is the total area of the farm?

23. A rectangular box is 4 ft, 3 in. long, 3 ft. 5 in. wide and 2 ft. 8 in. high. Find the total length of its twelve edges.

24. There are 52 sq. yd. 7 sq. ft. of plastering in the ceiling of a certain room, 30 sq. yd. 5 sq. ft. in each of the two side walls, and 23 sq. yd. 2 sq. ft. in each of the two end walls. Find the total area of plastering in the room.

25. Find the volume of four boxes measuring respectively 15 cu. ft. 1287 cu. in., 17 cu. ft. 1349 cu. in., 18 cu. ft. 793 cu. in. and 23 cu. ft. 978 cu. in.

IV. COMPOUND SUBTRACTION.

Compound Subtraction *is the operation of finding the difference between two similar compound numbers.*

EXERCISE IX.

1. How much must be added to 4 T. 1736 lb. to make the whole 19 T. 400 lb?

2. By how much is 10 gal. 2 qt. 1 pt. less than 20 gal. 2 qt.?

3. What weight added to 3 T. 1764 lb. 5 oz. will give the same weight as 5 T. 1943 lb. 8 oz. added to 1 T. 749 lb. 10 oz.

4. One week's supply of wheat to the Toronto markets was 14,750 bu. Of this quantity 8693 bu. 17 lb. came by rail, 4588 bu. 48 lb came by water, and the rest was bought from farmers from the surrounding country. How much was so bought?

5. Last year 19 mi. 400 yd. of pipe for water supply were in use in Hamilton; this year 22 mi. 100 yd. are in use. How much pipe has been laid during the year?

6. A crock of butter weighed 39 lb. 7 oz., and the crock weighed 6 lb. 12 oz. How much did the butter weigh?

7. What is the final remainder on taking 3 doz. and 5 as often as possible from 11 doz.?

8. A owes B $73.64; B owes A $29.33. B pays A $16.47, and A pays B $47.87. Which is indebted to the other, and how much?

9. I started out for a walk at 2 hr. 37 min. 43 sec. after noon, and got back exactly 5 hr. 9 min. after noon. How long had I been out?

10. How long is it from 24 min. 35 sec. past 8 in the morning to 12 min. 30 sec. past 4 in the afternoon?

11. A wheat buyer bought 196 bu. 48 lb. of wheat on Monday and 473 bu. 35 lb. on Tuesday, but sold 600 bu. on the latter day; he bought 847 bu. 19 lb. on Thursday, and 1573 bu. 40 lb. on Saturday, but sold 2000 bu. 39 lb. on the latter day. He had 169 bu. 15 lb. on hand at the beginning of the week. How much had he at the close?

12. Take a million inches from a hundred miles.

13. Into a barrel which would hold just 30 gal. there were poured 19 gal. 1 pt. of vinegar and 2 gal. 1 qt. of acetic acid, and the barrel was then filled up with water. How much water was poured in?

14. A farmer had 724 bu. of oats. He sold 429 bu. 1 pk. and fed to his horses 93 bu. 2 pk. 1 gal. 1 qt. How much had he left?

15. Three piles of wood contained respectively 12 cords 72 cu. ft., 27 cords 43 cu. ft., and 31 cords 96 cu. ft. There was sold from them 57 cords 100 cu. ft. What quantity remained?

16. A farm of 110 A. 2319 sq. yd. consists partly of woodland and partly of cleared fields. The cleared fields cover an area of 63 A. 3630 sq. yd. What is the area of the woodland?

17. A man had a farm measuring 125 A. 2420 sq. yd., of which 88 A. 3327 sq. yd. was cleared, the rest being in woodland. He sold 31 A. 2946 sq. yd. of the cleared land, and 7 A. 1327 sq. yd. of the woodland. How many acres of cleared land and how many of woodland had he left?

18. St. Paul's Cathedral, in London, England, is in latitude 51° 30′ 48″; St. Peter's, in Rome, Italy, is in latitude 41° 53′ 54″. What is the difference in their latitudes?

19. What is the difference in latitude and longitude between Madrid in 40° 24′ 35″ N. Lat. and 3° 41′ 51″ W. Long. and Montreal in 45° 31′ 27″ N. Lat. and 73° 32′ 30″ W. Long.?

20. Berlin is in 52° 30′ 16″ N. Lat.; Toronto is in 43° 31′ 45″ N. Lat. How much farther north is Berlin than Toronto?

21. A farmer had 75 cords of wood for sale. He sold at different times 7 cords 48 cu. ft., 15 cords 36 cu. ft., 5 cords 60 cu. ft., 18 cords 96 cu. ft., and 25 cords 64 cu. ft. How much had he still for sale?

22. A coal dealer agreed to deliver 22 T. 1000 lb. between the 1st July and the 1st September. He delivered 13 T. 1749 lb. in July. How much had he to deliver in August?

23. A sold to B on 3rd March goods amounting to $15.48, on 19th March goods amounting to $37.74, on 7th April goods amounting to $28.63, and on 28th April goods amounting to $45.63. B paid to A on 19th March $31.40, on 18th April $23.65, and on 1st May $50. How much did B still owe A after the last payment?

24. A merchant's accounts showed for July: receipts, $1746; expenditure, $1423.47. How much more did he receive than expend?

25. I sold goods for $97.48, gaining thereon $19.50. How much did the goods cost me?

26. Out of a cistern containing 1000 gal. of water 100 cu. ft. of water were drawn. Find the weight of the water remaining in the cistern.

V. COMPOUND MULTIPLICATION.

Compound Multiplication *is the operation of finding the product of two numbers, one of which, the multiplicand, is compound.*

EXERCISE X.

1. 7 lb. 5 oz. × 3.
2. 18 lb. 9 oz. × 4.
3. 3 gal. 2 qt. × 5.
4. 5 ft. 7 in. × 6.
5. 9 da. 13 hr. × 7.
6. 38 bu. 3 pk. 3 qt. × 49.
7. 47 da. 18 hr. 36 min. × 81.
8. 5 da. 13 min. 7 sec. × 100.
9. 7 mi. 1140 yd. × 23.
10. 3 mi. 811 yd. 2 ft. 6 in. × 2.

11. How much wood is there in three piles, each containing 17 cords 56 cu. ft.?

12. A farmer plowed 1 A. 1512 sq. yd. a day for 6 days. How much did he plow during the whole six days?

13. A boy gathered 1 pk. 3 qt. of berries each day for 5 days. How much did he gather altogether?

14. A grocer bought 166 lb. of butter at 18 cents a lb. He sold 148 lb. of it at 23 ct. a lb., and the rest of it at 12 ct. a lb. How much did he gain on the whole?

15. What distance will a wheel 12 ft. 10 in. in circumference roll in 999 revolutions?

16. A sulky wheel 14 ft. 8 in. in circumference made 3600 revolutions in an hour. What distance did the sulky go during the hour?

17. A car wheel 9 ft. 2 in. in circumference is rolling at the rate of 5 revolutions per second. How far does it go per hour?

18. A boy walks 1 mi. 163 yd. to school each morning, and the same distance home each afternoon on 211 days in each year for 5 years. How far does he thus walk during that time?

19. If 9 mi. 926 yd. 1 ft. 3 in. be taken *four* times from a certain quantity, there will still be 3 mi. 754 yd. 2 ft. 1 in. left. Find the quantity.

20. A farm of 97 A. 3347 sq. yd. is divided into four fields. The first contains 9 A. 1981 sq. yd.; the second contains three times as much as the first, and the third five times as much as the first. How much does the fourth field contain?

21. A drover bought 27 sheep at $3.65 each, and 36 others at $4.12 each. How much will he gain by selling them all at $4.37 each?

22. A merchant bought 24 pieces of cloth measuring 36 yd. each, at $18.72 a piece, and sold the whole at $1.07 a yard. How much did he gain on the whole?

23. How much coal oil is contained in 30 barrels, each containing 30 gal. 1 qt. 1 pt.?

24. A woman sells a grocer 23 lb. of butter at 19 ct. a lb., 63 lb. of cheese at 9 ct. a lb. and 13 doz. eggs at 14 ct. a doz.; and buys from him 3 lb. of tea at 55 ct. a lb., 12 lb. of sugar at 9 ct. a lb., 2 gal. molasses at 23 ct. a qt., 8 lb. of currants at 8 ct. a lb., 11 lb. of raisins at 13 ct. a lb., and 3 doz. oranges at 23 ct. a doz. The difference between what the woman bought and what she sold was paid in money. How much was it? Which had to pay it—the woman or the grocer?

25. The fore quarters of a lamb weighed 5 lb. 3 oz. each, and the hind quarters 7 lb. 5 oz. each. How much did the lamb weigh?

26. What is the capacity of a cistern that holds 127 pailfuls of 2 gal. 1 qt. each?

27. The average weight of 59 barrels of pork was 196 lb. 12 oz. The full weight of each barrel should have been 200 lb. How much did the fifty-nine barrels lack of full weight?

28. A man travels 97 mi. 550 yd. a day for 25 days. How far must he travel the 26th day in order to have travelled 2500 miles in all?

29. A room is 18 ft. 8 in. long and 13 ft. 5. in wide. What is the length around it?

30. A box is 3 ft. 4 in. long and 2 ft. 3 in wide. What length of string would go five times around it?

31. A farmer had 21 bags of wheat, each containing 2 bu. 18 lb. How much had he in all?

32. The furnaces of a certain steamer burn 3 cords 72 cu. ft. of wood daily. How much wood will they burn in 95 days?

33. A merchant paid $49 for 7 bbl. of cranberries, containing 2 bu. 3 pk. 1 qt. each, and retailed them at 10 ct. a pint. How much did he gain on the whole?

34. A watch gains 1 min. 7 sec. per day. How much will it gain in a fortnight?

35. What is the length of 144 rails, each 16 ft. 6 in. long?

36. In a certain voyage a steamer averaged 14 mi. 513 yd. 1 ft. 6 in. per hour for 9 days. What was the distance run in that time?

VI. COMPOUND DIVISION.

Compound Division *is the operation of finding the quotient when the dividend, the divisor, or both of them, are compound numbers.*

CASE I.—WHEN THE DIVISOR IS AN ABSTRACT NUMBER.

EXERCISE XI.

1. 6 lb. 12 oz. ÷ 2.
2. 7 lb. 5 oz. ÷ 3.
3. 15 T. 156 lb. ÷ 4.
4. 19 T. 378 lb. 2 oz. ÷ 5.
5. 28 gal. 2 qt. ÷ 6.
6. 31 gal. 2 qt. ÷ 7.
7. 745 bu. 3 pk. ÷ 8.
8. 426 bu. 3 pk. 6 qt. ÷ 9.
9. 29 da. 7 hr. 37 min. ÷ 7.
10. 42 hr. 56 min. 24 sec. ÷ 9.
11. 97° 37′ 36″ ÷ 4.
12. $73.26 ÷ 9.
13. $183 ÷ 4.
14. 19 mi. 1354 yd. ÷ 6.
15. 129 mi. 1030 yd. 1 ft. 6 in. ÷ 7.
16. 193 mi. 1467 yd. ÷ 9.
17. 49 mi. 649 yd. 6 in. ÷ 5.
18. 47 cu. yd. 11 cu. ft. ÷ 3.
19. 104 cu. yd. 5 cu. ft. ÷ 9.
20. 48 A. 7 sq. ch. 2464 sq. l. ÷ 8.
21. 10 A. 1343 sq. yd. ÷ 3.
22. 497 A. 2714 sq. yd. ÷ 9.

23. Twelve boys gathered 11 bu. 2 qt. of nuts and divided them equally among themselves. How much did each boy receive?

24. If 11 men can mow 24 A. 968 sq. yd. of grass in a day, how much can one man mow?

25. From the half of 21 cu. yd. 21 cu. ft. take 2 cu. yd. 12 cu. ft., and divide the remainder into 12 equal parts.

26. If a stonemason lays 33 cu. yd. 3 cu. ft. of stone in 6 days, how much does he lay per day?

27. If 97 bushels of wheat be divided into six equal parts and three of these given to A, two to B, and one to C, what quantity will each get?

28. If a farm of 100 acres be divided into 9 equal-sized fields, what will be the area of each?

29. A piece of land measuring 56 A. 3817 sq. yd. is divided off into 33 equal allotments. What is the size of each?

30. A farm of 100 acres is surveyed off as a village site. One-eighth of the whole is laid out as streets, and the remainder is divided into 160 lots of equal size. What is the size of each lot?

31. Seven horses eat 13 bu. 3 pk. 1 qt. of oats in a week. What quantity does each horse eat per week?

CASE II.—WHEN THE DIVISOR IS A CONCRETE NUMBER.

In this case the divisor and the dividend must be quantities of the same kind; the quotient will be an abstract number expressing how many times the dividend contains the divisor.

EXERCISE XII.

1. 2 lb. 8 oz. ÷ 4 oz. *3.* 15 lb. 10 oz. ÷ 1 lb. 9 oz.

2. 10 lb. 8 oz. ÷ 12 oz. *4.* 725 lb. 5 oz. ÷ 3 lb. 7 oz.

5. 137 T. 1189 lb. 4 oz. ÷ 304 lb. 12 oz.

6. 15 da. 18 hr. ÷ 9 hr. *7.* 2 da. 2 hr. ÷ 50 min.

8. 6 da. 6 hr. 20 sec. ÷ 1 min. 5 sec.

9. 13 wk. 1 da. ÷ 3 hr. 50 min.

10. 12 gal. 1 qt. 1 pt. ÷ 1 gal. 1 qt. 1 pt.

11. 4851 gal. ÷ 31 gal. 2 qt.

12. 119 bu. 2 pk. 1 qt. ÷ 1 pk. 1 qt.

13. 102,336 bu. 2 pk. 3 qt. 1 pt. ÷ 11 bu. 1 pk. 3 qt. 1 pt.

14. 8 yd. 2 in. ÷ 2 ft. 5 in. *15.* 1 mi. ÷ 2 ft. 6 in.

16. 3 mi. 550 yd. ÷ 2 ft. 9 in. *17.* 25 mi. 100 yd. ÷ 2 yd. 1 ft. 6 in.

18. 999 mi. 544 yd. 2 ft. 3 in. ÷ 10 mi. 418 yd.

19. 13,900 sq. yd. 2 sq. ft. 127 sq. in.
÷ 116 sq. yd. 7 sq. ft. 41 sq. in.

20. 1254 A. 2435 sq. yd. 2 sq. ft. 36 sq. in.
÷ 11 A. 3505 sq. yd. 6 sq. ft. 108 sq. in.

21. 64,447 A. 573 sq. yd. 7 sq. ft. 106 sq. in.
÷ 12 A. 4049 sq. yd. 8 sq. ft. 2 sq. in.

22. 1,764,578 cu. yd. 18 cu. ft. 1129 cu. in.
÷ 19 cu. yd. 11 cu. ft. 119 cu. in.

23. $14.50 ÷ $2.90. *24.* $1110 ÷ $3.70.

25. $1001 ÷ 13 ct.

26. How many yards of sateen at 15 ct. a yard can be purchased for $4.95?

27. How often can 77 sq. yd. be subtracted from 1 A. 3630 sq. yd.?

28. How many posts placed 7 ft. apart will be required to support a fence around a field, the length of the fence being 354 yd. 2 ft.? How many posts would have been required had the fence been straight?

29. How many sleepers laid 2 ft. 6 in. from centre to centre will be required for a railway 56 mi. 550 yd. long?

30. How many dress pieces each 15 yd. 18 in. long can be cut from a piece of goods 403 yd. long?

31. How long will 16 bu. 2 pk. of oats last a horse, giving him 3 feeds a day of 5 qt. 1 pt. each?

32. How long will 131 T. 100 lb. of food last 960 men, allowing them 1 lb. 4 oz. per day per man?

33. How many bars of lead each weighing 13 lb. 7 oz. will be required to make up a weight of 20 T. 1468 lb. 2 oz.?

34. How many loads of coal weighing 1 T. 80 lb. each are there in 13 car loads weighing 16 T. 1600 lb. each?

35. How many barrels holding 1 bu. 3 pk. 6 qt. each will a farmer require to pack 310 bushels of apples for market?

36. How many cans holding 4 gal. 1 qt. 1 pt. each can be filled out of 5 barrels containing 31 gal. 2 qt. each?

37. How long would a cannon ball travelling at the rate of 1320 ft. per second take to pass from the earth to the moon, a distance of 238,828 miles?

38. How many chains of 66 feet each would make 4 miles 880 yards?

39. How long would it take to walk 18 mi. at the rate of 108 steps of 2 ft. 8 in. each per minute?

40. If the average speed of an express train be 27 miles 609 yards per hour, how long will it take to travel 382 miles 1486 yards?

41. How many portions of time each equal to 1 da. 14 hr. 57 min. 33 sec. are contained in 365 da. 5 hr. 48 min. 45 sec.?

42. How many turns will a wheel 14 ft. 3 in. in circumference make in rolling a distance of 11 mi. 1559 yd.?

43. How many pieces of ribbon each 5 yd. 9 in. long can be cut from a ribbon 100 yd. long, and what length will remain over?

44. How many bottles each holding 1 qt. 1 pt. can be filled from a barrel containing 31 gal. 2 qt.?

45. A regiment in close column occupied 359 sq. yd. 5 sq. ft. 108 sq. in. How many men were there in the regiment if each man occupied 3 sq. ft. 52 sq. in.?

46. How long will it take to plough 50 A. 3025 sq. yd. at the rate of 4 A. 1058 sq. yd. 6 sq. ft. 108 sq. in. per day?

47. A farm of 265 A. was surveyed off into a village site. Of this area the streets required 27 A. 302 sq. yd. 4 sq. ft. 72 sq. in., and the rest was laid off into lots of 3877 sq. yd. 4 sq. ft. 72 sq. in. each. How many lots were there?

48. How many lunations of 29 da. 12 hr. 44 min. 3 sec. each are contained in 19 years of 365 da. 5 hr. 48 min. 45 sec. each, and by how much would one more lunation exceed the 19 years?

CHAPTER IV.

APPLICATIONS OF THE PRECEDING RULES.

I. VALUES.

The **Value** of anything is the amount of money for which it will sell, or the quantity of any other commodity for which it can be exchanged.

The **Cost** of anything is the amount of money paid for it, or the amount of any other commodity given in exchange for it.

The **Price** of any quantity of a commodity is the amount of *money* for which that quantity of the commodity is bought or sold or offered for sale.

The **Price-Rate** is the price per unit, or per other standard quantity, of the commodity.

The **Price-Unit** is the standard quantity of the commodity on which the price-rate is based.

EXERCISE XIII.

Find the price of—

1. 8 lb. of beef @ 12 ct. a lb. *2.* 17 yd. calico @ 13 ct. a yd.

3. 6 pair of chickens @ 65 ct. a pair.

4. 27 doz. eggs @ 17 ct. a doz.

5. 19 doz. clothes-pins @ 7 ct. a doz.

6. Two fish, the one weighing 9 lb., the other weighing 12 lb., both @ 14 ct. a lb.

7. Three crocks of butter weighing 27 lb., 25 lb. and 24 lb. respectively, all @ 19 ct. a lb.

8. 4 pair of chicken @ 55 ct. a pair, 3 pair of ducks @ 75 ct. a pair, 3 geese @ 65 ct. each, and 5 turkeys @ $1.05 each.

9. 7 lb. black tea @ 65 ct. a lb., 4 lb. coffee @ 35 ct. a lb., 7 lb. loaf sugar @ 12 ct. a lb., 8 lb. crushed sugar @ 9 ct. a lb., 8 lb. cheese @ 14 ct. a lb., and 13 lb. Carolina rice @ 9 ct. a lb.

10. 3 doz. handkerchiefs @ 45 ct. each.

11. 2 doz. tins of tomatoes @ 9 ct. each.

12. 5 doz. tins of sweet corn @ 11 ct. each.

13. 3 gal. 2 qt. of molasses @ 18 ct. a qt.

14. 4 lb. 7 oz. of rhubarb @ 25 ct. an oz.

15. 3 lb. 11 oz. of iodide of potassium @ 55 ct. an oz.

16. 5 lb. 5 oz. of quinine @ $2.25 an oz.

17. Find the cost of gravelling 3 mi. 330 yd. of road @ $8.50 a rod. (2 rods = 11 yards.)

18. Find the cost of 48 rods of fencing @ 75 ct. a yd.

Find the value of—

19. A steam-hammer weighing 189 T. @ 28 lb. to the dollar.

20. 60 packages of dried yeast @ 20 ct. a doz.

21. One million bricks @ $7.75 a M.

22. 587,000 ft. of lumber @ 15.75 a M.

23. 1380 lb. of wheat @ 87 ct. a bu.

24. 1470 lb. " @ 98 ct. "

25. 6480 lb. " @ $1.17 "

26. 1938 lb. of oats @ 37 ct. "

27. 2346 lb. " @ 45 ct. "

28. 9486 lb. " @ 39 ct. "

29. 1872 lb. of barley @ 57 ct. "

30. 2832 lb. " @ 63 ct. "

31. 47,856 lb. " @ 59 ct. "

32. 1620 lb. of peas @ 77 ct. "

33. 2340 lb. " @ 69 ct. "

34. 40,680 lb. " @ 57 ct. "

35. 1512 lb. of rye @ 68 ct. "

36. 2464 lb. " @ 63 ct. "

37. 2744 lb. of Indian corn @ 57 ct. a bu.

38. 51,464 lb. " " @ 49 ct. "

39. 630 lb. of bituminous coal @ 32 ct. a bu.

40. 1740 lb. of carrots at 17 ct. a bu.

41. 8 burners consuming 5 cubic feet of gas each per hour are used at the rate of 5 hr. a day for 310 days. Find the cost of the gas burned @ $2.25 per 1000 cu. ft.

42. What will be the amount of a man's wages for 6 days of 9 hr. each @ 18 ct. an hour?

43. How much will a man earn in three weeks @ $2.25 a day, omitting Sundays?

44. A man's wages are $2.25 a day of 10 hr. and 35 ct. an hour for over-time. How much ought he to receive for 16 full days and 25 hr. over-time?

45. A mechanic receives $2.70 a day of 10 hr. and 45 ct. an hour for over-time. What were his wages for a week on which he worked: Monday, 11 hr.; Tuesday, 13 hr.; Wednesday, 10 hr.; Thursday, 12 hr.; Friday, 10 hr.; Saturday, 14 hr.?

46. A man worked from 1st September to 19th October, both days included, @ $1.90 a day, Sundays omitted. How much did he earn? Out of this he paid 30 ct. a day for board (Sundays included); how much had he left?

47. A man received $1.75 a day, omitting Sundays. What was the amount of his wages for February, 1888? (February, 1888, commenced on a Wednesday.)

48. Find the cost of hauling 37 T. 12 cwt. a distance of 19 miles @ 3 ct. per cwt. per mile.

49. Find the expenses of 7 persons for a journey of 261 miles, the railway fare being 3 ct. each per mile, and the other expenses amounting to $2.50 each?

50. If 1,081,130 persons in Canada were to send by post 37 letters each on an average during a year, find what the postage on these would amount to per day, the postage averaging 3 ct. on each letter.

51. A merchant sells 13 yd. of calico @ 12 ct. a yd., 19 yd. of muslin @ 23 ct. a yd., and 17 yd. of flannel @ 48 ct. a yd., and takes in exchange 38 bu. of potatoes @ 37 ct. a bu. and the balance in cash. How much cash does he receive?

52. A farmer sells to a grocer 19 doz. of eggs @ 18 ct. a doz., 47 lb. of lard @ 13 ct. a lb., and 117 lb. of beef @ 8 ct. a lb., and takes in exchange 7 lb. of tea @ 55 ct. a lb., 9 lb. of coffee @ 35 ct. a lb., a set of dishes worth $7.50, and the balance in cash. How much cash is due him?

53. A woman sells to a grocer 15 doz. eggs @ 18 ct. a doz. and 35 lb. of butter @ 23 ct. a lb., and buys from him 9 lb. of tea @ 55 ct. a lb., 15 lb. of sugar @ 7 ct. a lb., 5 bars of soap @ 25 ct. a bar, 1 gal of vinegar @ 16 ct. a qt., 4 lb. of raisins @ 12 ct. a lb., 4 lb. of currants @ 7 ct. a lb., 2 oz. of cinnamon @ 3 ct. an oz., and 8 oz. of allspice @ 3 ct. an oz. How much is still due her? If she receive this sum in 5-cent pieces, how many ought she to get?

54. A woman sells a merchant 7 pair of chickens @ 56 ct. a pair, 5 pair of ducks @ 73 ct. a pair, 4 geese @ 93 ct. each, and 3 turkeys @ $1.15 each, and buys from him 13 yd. of calico @ 15 ct. a yd., 19 yd. of calico @ 17 ct. a yd., 17 yd. of flannel @ 45 ct. a yd., and 29 yd. of chintz @ 26 ct. a yd. How much is due the merchant on this transaction?

55. A farmer sells 2220 lb. of wheat @ 98 ct. a bu. and 2686 lb. of oats @ 37 ct. a bu., and with the proceeds he buys 49 yd. of carpet @ $1.15 a yard and 24 rolls of wall paper @ 37 ct. a roll. How much has he left of the cash received for his sales?

56. How many yards of cloth @ $1.55 a yd. can be bought for $26.35?

57. If 10 yd. of cloth cost $14.50, how many yards can be bought for $27.55?

58. How many yards of cloth @ $1.35 a yd. can be bought for 30 bu. of wheat @ 90 ct. a bu.?

59. If 7 yd. of cloth cost $8.40, how many yards ought to be received for 15 bu. of wheat @ 96 ct. a bu.?

60. If 7 bu. of oats are worth 3 bu. of wheat, how many bu. of oats are worth 51 bu. of wheat?

61. If 13 bu. of barley are worth 9 bu. of wheat, how many bu. of barley are worth 1620 lb. of wheat?

62. If 5 bu. of oats are worth 2 bu. of wheat, how many pounds of wheat should be given for 1870 lb. of oats?

63. How many sheep at 3 for $13 can I buy for $117?

64. How many hogs at 7 for $48 can I buy with $700 and have $28 left?

65. If a man receive 9 lb. of tea in exchange for 45 lb. of cheese @ 11 ct. a lb., what is the price of the tea per pound?

66. A woman sold 27 lb. of butter @ 23 ct. a lb., and bought 13 lb. of sugar @ 7 ct. a lb. and 4 lb. of coffee @ 35 ct. a lb. How many pounds of tea @ 65 ct. a lb. could she buy with what was still left of the amount she got for the butter?

67. A farmer gave 85 bu. of wheat, worth $1.18 a bu., for 10 sheep. How much apiece did the sheep cost him?

68. A mechanic earns $60 a month, but his expenses are $45 a month. How long will it take him to pay for a farm of 80 acres worth $36 an acre?

69. A newsboy buys 7 doz. newspapers @ 20 ct. a doz. and sells them @ 3 ct. a paper. If he sell all but 5 papers, how much will he gain?

70. An apple-woman bought 9 doz. apples @ 15 ct. a doz., and sold 24 of the apples @ 2 for 3 ct. and the remainder @ 2 ct. an apple. How many dozen apples @ 17 ct. a doz. can she buy with the proceeds?

II. BILLS AND ACCOUNTS.

A **Bill of Parcels** (called also a *Bill of Goods*) is a written statement of goods sold and of payments, if any, received therefor. The Bill should specify the quantities and prices of the goods, the place and time of each transaction, the names of the buyer and the seller, and any special terms agreed on by the parties.

A **Bill of Services** is a similar statement of services rendered or of labor performed.

A Bill is also called an **Account.**

A **Statement of Account** is a written statement of the total sums due according to accounts already rendered.

The seller of the goods is called the **Creditor.**

The buyer of the goods is called the **Debtor.**

The statement of the items due to the party rendering the account is called the **Debit Side of the Account.**

The statement of the items due or the moneys received by the party rendering the account is called the **Credit Side of the Account.**

The **Balance** of an account is the difference between its debit and credit sides.

When a bill is paid it should be receipted by writing at the bottom of the bill the date of payment and the words "Received payment," and under these words the creditor should sign his name.

If a clerk or other employé have authority to sign for his employer, he should write his employer's name and directly beneath it his own name or initials, preceded by *per* or *by*. He may, instead of signing in this way, write his own name and directly beneath it his employer's name, preceded by *for*.

EXERCISE XIV.

Make out Bills for the following-mentioned transactions, supplying, where necessary, names of places and dates of making out:—

1. Mr. James Thompson bought of William Smith: Nov. 2nd, 1885, 14 yd. Print @ 13 ct. a yd.; Nov. 3rd, 33 yd. White Cotton @ 14 ct.; Nov. 6th, 17 yd. Tweed @ $1.18; Nov. 12th, 16 yd. Silk @ $1.87, 9 yd. Lining @ 13 ct., and 3 doz. Buttons @ 23 ct. a doz.; Nov. 14th, 9 yd. Jersey Cloth @ 45 ct., 2 yd. Plush @ $1.95, 3 yd. Skirt Lining @ 18 ct., 2 doz. Buttons @ 15 ct., 2 Spools @ 5 ct.

2. Mr. H. Williams bought of Thos. Acroyd: Nov. 19th, 1885, 9 lb. Roast Beef @ 12 ct.; Nov. 21st, 7 lb. Lamb @ 13 ct., 4 lb. Suet, @ 9 ct.; Nov. 23rd, 8 lb. Bl. Beef @ 8 ct.; Nov. 25th, 3 lb. Steak @ 14 ct.; Nov. 26th, 13 lb. Lamb @ 13 ct.; Nov. 28th, 5 lb. Corned Beef @ 9 ct. and 2 Geese @ 65 ct. each; Dec. 1st, 3 pair of Chickens @ 55 ct. a pair; Dec. 3rd, 6 lb. Venison @ 14 ct. and 9 lb. Sausages @ 12 ct.

3. George Beardwood sold to Thos. Conroy on 26th Sept., 1885, 14 cords Maple @ $3.50, 4 cords Soft Wood @ $2.25, and 7 cords Sawn Hardwood @ $4.25.

4. Benj. Bradshaw bought of John Westover on 7th Jan., 1886, 700 lb. of Flour @ $2.75 a cwt., 400 lb. Oatmeal @ $2.25, 300 lb. Cornmeal, @ $2.25, and 200 lb. Buckwheat Flour @ $2.50. On 1st Feb., 1886, Benj. Bradshaw paid $25 on the above account.

5. William Atkinson bought of Messrs. Moore & Co., Ap. 8th, 1886, 100 ft. ¾-in. 3-ply Rubber Hose @ 20 ct. a ft., 2 pr. ¾-in. Couplings & fittings @ 50 ct. a pr., 1 Comp. Hose Pipe, $1.25; Ap. 16th, 3 Step-Ladders @ $1.50 each. The above account was paid in full on Ap. 16th.

6. Messrs. Hughes & Son sold to M. Stonehouse: Dec. 9th, 1885, 19 yd. Calico @ 17 ct., 17 yd. Linen @ 47 ct., 16 yd. Lining @ 9 ct.; Dec. 21st, 8 yd. Flannel @ 48 ct., 23 yd. Braid @ 3 ct.; Dec. 26th, 7 pr. Stockings @ 25 ct. and 3 pr. Gloves, @ 65 ct. Paid in full on 2nd Jan., 1886.

7. Peter Simpson bought of Jamieson Bros.: 14th Ap., 1885, 3 lb. Bl. Tea @ 75 ct. and 13 lb. B. L. Sugar @ 11 ct.; Ap. 16th, 5 lb. Gran. Sugar @ 9 ct.; Ap. 18th, 3 Bars Soap @ 23 ct., 3 boxes Starch @ 15 ct.; Ap. 21st, 1 Bath-brick @ 8 ct., 3 doz. Eggs @ 17 ct., and 4 lb. Butter @ 19 ct.; Ap. 23rd, 12 lb. Flour, @ 3 ct., 1 box Soda Biscuits @ 30 ct.; Ap. 25th, 4 lb. Currants @ 8 ct. and 7 lb. Raisins @ 9 ct. On Ap. 21st the sum of $5 was paid on the above account, and the balance was paid on 1st May.

8. Edward Lawson bought of Bruce, Playfair & Co. : 5th Jan., 1886, 9 Diaries @ 57 ct., 3 boxes Elastic Bands @ 25 ct., 5 Rms. F'scap @ $3.45; Jan. 13th, 3 qr. Blotting Paper @ 37 ct., 7 boxes Pens @ 36 ct., 5 boxes Slate-pencils @ 17 ct. ; Jan. 22nd, 16 doz. 6×9 Slates @ 95 ct. ; Jan. 25th, 5 qt. Ink @ 37 ct., 5 qr. Wrapping Paper @ 30 ct., 6 Col. Pencils @ 9 ct.; Feb. 3rd, 2 Rm. Acct. Cap @ $6.00; 1 Rm. Letter Paper @ $4.50, 3 Pass-Books @ 5 ct. ; Feb. 11th, 3 boxes Envelopes @ $1.25, Postage stamps, $4. On this account the sum of $15 was paid on Jan. 16th, and a further sum of $25 was paid on Feb. 15th.

9. Simon Tomlinson bought of H. Ward & Co., of Guelph, in 1885: July 4th, 5 doz. Hat and Coat Hooks @ 40 ct., 3 Door Knobs @ 15 ct., and 3 Rack Pulleys @ 20 ct.; 7th, 25 lb. Cut Nails @ 4 ct., 3 pr. Hinges @ 23 ct., and 2 Door Locks @ 30 ct. ; 18th, 7 lb. Pressed Nails @ 8 ct., 9 doz. Screws @ 6 ct.; Aug. 1st, 3 Padlocks @ 25 ct., 3 Hasps and Staples @ 15 ct.; 2 doz. Bolts @ 20 ct. ; 20th, 5 lb. S. L. Cord @ 90 ct. ; 5 yd. Brass Chain @ 33 ct. Aug. 29th, Account paid in full.

10. George Stevens owed Samuel Crearar on 30th Nov., 1885, as per account rendered that day, the sum of $14.92; and during December, 1885, he bought of Sm. Crearar as follows :—1st, 8 lb. Ro. Beef @ 12 ct. ; 3rd, 6 lb. Leg Lamb @ 11 ct. ; 2 lb. Vn. Steak @ 14 ct., and 1 lb. Suet @ 12 ct. ; 8th, 12 lb. Ham @ 15 ct., 9 lb. Corned Beef @ 9 ct.; 10th, 2 lb. Steak @ 13 ct., 1 Beef Heart, 25 ct.; 12th, 5 lb. Loin Veal @ 11 ct., 1 pr. Fowl, 70 ct., 1 lb. Suet @ 13 ct.; 16th, 11 lb. Hind Qur. Lamb @ 13 ct., 10 lb. Ro. Beef @ 12 ct.; 18th, 2 Tongues @ 45 ct., 7 lb. Shank @ 4 ct. ; 2 lb. Lard. @ 15 ct.; 22nd, 1 Goose, 80 ct., 1 Turkey, $3.50; 23d, 1 Corned Tongue, 50 ct., 3 lb. Steak @ 13 ct., and 1 lb. Suet @ 12 ct. ; 28th, 11 lb. Ro. Beef @ 13 ct.; 30th, 6 lb. Leg Lamb @ 14 ct., 4 lb. Lard @ 15 ct. On this account there was paid $15 on Dec. 3rd, and the balance in full on 31st Dec.

11. Mrs. G. Scott bought of Edwin Salter, Grocer, Belleville, during Feb., 1886, as follows :—1st Feb., 9 lb. Bl. Tea @ 70 ct., 5 lb. Java Coffee @ 34 ct., 9 lb. Sugar @ 11 ct., 8 lb. sugar @ 7 ct.; 5th, 3 lb. Raisins @ 9 ct., 7 lb Currants @ 6 ct., 3 lb. Figs @ 11 ct.; 8th, 4 Bars Soap @ 25 ct., 1 Box Starch, 15 ct.; 13th, 3 doz. Ale @ $1.15, 1 Tin Marmalade, $1.30; 17th, 1 Crock Butter, 23 lb., @ 19 ct. ; 22nd, 2 lb. Japan Tea @ 60 ct., 3 lb. Cheese @ 15 ct.; 27th, 2 lb Arrowroot @ 25 ct., 2 doz. Bloaters @ 35 ct., 22 lb. Beef Ham @ 16 ct. This account was made up on 1st March, 1886. On the 5th March credit was given on it for 3 doz. Ale @ $1.15, wrongly charged on Feb. 13th, and the balance was then paid in full.

III. AGGREGATES AND AVERAGES.

The **Total** or **Aggregate** of any number of quantities of the same kind is simply their *sum*. Hence—

To find the Total or Aggregate of any number of quantities of the same kind, add the quantities together.

The **Average** or **Mean** of any number of quantities of the same kind is that quantity which, if put in place of each of the given quantities, will yield a sum the same as that of these quantities. Hence—

To find the Average or Mean of any number of quantities of the same kind, divide the sum of the quantities by the number of them.

EXERCISE XV.

Complete the following tabulated statements by filling in the totals, and, where they occur, the columns of differences.

1. Classification of pupils, Cities of Ontario, 1883.

CITY.	NUMBER OF PUPILS IN THE						TOTAL.
	1st Class.	2nd Class.	3rd Class.	4th Class.	5th Class.	6th Class.	
Belleville	1024	488	526	263	14	....	
Brantford.....	782	454	772	374	. ..	....	
Guelph..	727	366	648	415	....	....	
Hamilton.....	3553	1613	2055	916	104	16	
Kingston	1216	672	841	560	240	120	
London.....	1681	1108	1389	564	51	....	
Ottawa	1753	1367	1550	558	402	19	
St. Catharines.	810	489	569	439	87	4	
St. Thomas	1068	523	359	347	....	....	
Toronto	7466	4253	3742	1940	849	158	
Total							

2. Statement of school attendance during the year 1884.

TOWNSHIP	NUMBER OF PUPILS WHO ATTENDED						TOTAL.
	Less than 20 da.	From 20 to 50 da.	From 51 to 100 da.	From 101 to 150 da.	From 151 to 200 da.	From 201 to 222 da.	
Charlottenb'gh	108	255	336	287	171	42	
Kenyon.....	121	256	315	280	195	25	
Lancaster.. ...	89	218	317	255	179	32	
Lochiel..	105	175	233	276	136	20	
Total, 1884. ..							
Total, 1883. ..	528	871	1207	963	765	162	
Increase.....							
Decrease.....							

3. Statement of Canadian live stock, 1881.

PROVINCES.	Horses.	Cattle.	Sheep.	Swine.	TOTAL.
Prince Edward Is.	31,335	90,722	166,496	40,181	
Nova Scotia	57,167	325,603	377,801	47.256	
New Brunswick ..	52,975	212,565	221,163	53,087	
Quebec.	273,852	1,030,333	889,833	329,199	
Ontario.....	590,298	1,702,167	1,359,178	700,922	
Manitoba.....	16,739	60,281	6,073	17,358	
British Columbia .	26,122	80,451	27,788	16,841	
Territories.....	10,870	12,872	346	2,775	
Total.					

4. School Trustees' Financial Statement.

TOWNSHIP.	Balance from 1883.	Receipts during 1884	Total Receipts.	Expendit're during 1884.	Balance on hand.
Charlottenb'gh	$593 96	$6116 15		$6154 39	
Kenyon	194 57	6294 91		5917 53	
Lancaster	546 37	6527 74		6230 89	
Lochiel	396 78	4482 99		3961 62	
Total..					

5. What is the mean of 3 and 7? Of 5 and 11? Of 7 and 25? Of 10 and 20? Of 0 and 100?

6. What is the mean of 2, 5 and 11? Of 3, 6 and 12? Of 5, 7 and 9? Of 0, 8 and 10? Of 2, 2 and 20?

7. What is the average of two weights of 3 lb. and 4 lb. respectively? Of 9 lb. and 29 lb. respectively? Of 1 lb. and 10 lb. respectively?

8. What is the average of three lengths respectively of 4 ft., 5 ft. and 7 ft.? Of 10 ft., 25 ft. and 50 ft.?

9. What is the average of 4 weights of 7 lb., 10 lb., 15 lb. and 19 lb. respectively?

10. What is the average of four lengths of 5 ft., 10 ft., 20 ft. and 40 ft. respectively?

11. Four vessels holding respectively 2 gal. 2 qt., 1 gal. 1 qt., 3 gal., and 2 gal. 3 qt., are full of water. How large a vessel would this water fill four times?

12. What is the average of 4 loads of wheat of 36 bu., 29 bu. 40 lb., 33 bu. 30 lb. and 35 bu. 10 lb. respectively?

13. Find the average of 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

14. Find the average of the following scores at a rifle match: 81, 79, 79, 75, 72, 72, 68, and 66.

15. Find the average of the following scores at cricket: 49, 36, 54, 8, 0, 9, 17, 8, 4, 2, 0.

16. A grocer's daily receipts were: Monday, $219.57; Tuesday, $247.38; Wednesday, $213.45; Thursday, $368.72; Friday, $245.19; Saturday, $473.77. Find his average daily receipts for the week.

17. The daily receipts of a hardware merchant were: Monday, $473.67; Tuesday, $594.68; Wednesday, $371.93; Thursday, (a holiday), nothing; Friday, $687.55; Saturday, $749.47. Find his average daily receipts—1st, *excluding* Thursday; 2nd, *including* Thursday.

18. The monthly sales of a merchant were: January, $4378.46; February, $3753.69; March, $5685.75; April, $4293.38. Find the average sales per month for the four months. If the same average rate had continued throughout the year, what would have been the total amount of his sales that year?

19. If a man spend $142.31 in 19 weeks, how much does he spend on an average per week? At that rate how much would he spend in a year (52 wk. 1 da.)?

20. Eight boys weigh respectively 109 lb., 105 lb., 103 lb., 97 lb., 111 lb., 88 lb., 106 lb., and 102 lb. What is their average weight?

21. If a dozen eggs weigh 1 lb. 8 oz., what will be the average weight of an egg?

22. A man walked 7 miles one day, 9 miles the second, 8 miles the third day, and 10 miles the fourth day. What was the total distance he walked and the average distance per day?

23. John is 12 years old, his sister is 10, his eldest brother is 15, and his youngest brother is 7. What is the aggregate and what the average of their ages?

24. A grocer bought a tub of butter weighing 34 lb. @ 18 ct. a lb., a second tub weighing 42 lb. @ 19 ct., a third tub weighing 48 lb. @ 21 ct., and a fourth tub weighing 31 lb. @ 22 ct. What was the total weight and price of the four tubs, and what the average price per pound?

25. In a Third Class there were five boys whose respective heights were 4 ft. 8 in., 4 ft. 10 in., 4 ft. 7 in., 4 ft. 6 in., and 4 ft. 9 in. What was the aggregate of their heights and what their average height?

26. A man worked 10 hours on Monday, 8 on Tuesday, 9 on Wednesday, 7 on Thursday, 9 on Friday, and 8 on Saturday. What was the total and what the daily average time he worked during the week?

27. If 19 hams weigh 276 lb. 11 oz., what is their average weight?

28. A man earns $1365 a year. How much is that on an average a week, taking 52 wk. = 1 year?

29. An express company carries 30,553 T. 604 lb. of merchandise in the course of a year. What is the average weight carried per week? (Take 52 weeks for a year.)

30. Smith's wages are $1.60 a day; Brown's are $1.75; Jones' $2.10; Robinson's $2.40; and Thompson's $2.25. What is the aggregate and what the average of their daily wages?

31. If a man's salary be $1250, how much may he spend on an average per day, and how much per week, to the nearest cent, so as not to run into debt? (Reckon 52 weeks to the year, also 365 days to the year.)

32. The total weight of 17 cheese was 929 lb. 11 oz. What was their average weight?

33. A man walked 373 yd. 1 ft. in 480 steps. What was the average length of his steps?

34. A man walked 500 miles in 24 days. How far did he walk on an average per day?

35. A traveller left New York by the Pacific Express at 10 o'clock on Tuesday morning, and arrived at San Francisco at 11 o'clock, New York time, on the following Monday morning, having travelled a distance of 3364 miles. At what average rate per hour did he travel?

36. If 11 men have to mow 24 A. 968 sq. yd. of grass in 11 hr., how much must each man mow on an average per hour?

37. A wall containing 412 cu. yd. of stone was built in 6 weeks. What was the average amount built per day (6 working days to the week)?

38. Five men took turns to keep watch over a house for 13 da. 19 hr. If each man kept watch thirty times, what was the average length of each watch?

39. The following summary is taken from a book of cash sales :—

		Amount.
Aug. 7,	sold 310 @ $1.09 each	
8,	" 470 @ 1.25 "	
9,	" 640 @ .95 "	
10,	" 430 @ 1.07 "	
11,	" 580 @ .99 "	
12,	" 360 @ 1.16 "	

What was the average number sold daily, the average daily cash business, and the average selling price?

40. A man has a salary of $850 a year; for the first 7 months of a certain year he spent an average of $85 a month. How much can he spend a month for the remainder of the year and not live beyond his salary?

41. A grocer mixes together 40 lb. of tea @ 45 ct. a lb., 48 lb. @ 47 ct., and 64 lb. @ 53 ct. What is the price per lb. of the mixture?

42. A mixture was made of three grades of barley, viz., 8 bu. @ 59 ct., 15 bu. @ 58 ct., and 28 bu. @ 65 ct. What is the value per bushel of the mixture?

43. A stationer bought 72 reams of paper @ $3.60 a ream, and 48 reams @ $6.60 a ream. Find the cost of the whole, and the average price per quire and per sheet.

44. A grocer mixed 106 lb. of tea costing 38 ct. a lb., 75 lb. costing 42 ct. a lb., and 94 lb. costing 45 ct. a lb., and sold the mixture at 60 ct. a lb. What was his gain on the whole?

45. A grocer mixed 19 lb. of coffee costing 28 ct. a lb. and 26 lb. costing 23 ct. a lb. with 10 lb. of chickory costing 8 ct. a lb. At what price per lb. must he sell the mixture to gain $5.50 on the whole?

46. Find the total value and the value per gal. of a mixture of 17 gal. of vinegar @ 60 ct., 27 gal. of vinegar @ 40 ct., and 6 gal. of water.

47. How much water must be added to a mixture of 16 qt. of vinegar @ 13 ct. and 10 qt. at 10 ct.. that the whole mixture may be worth 11 ct. a qt.?

48. A barrel of vinegar containing 25 gal. was bought for $9. How much water had to be added to allow the mixture to be sold without loss @ 25 ct. a gal.?

49. A barrel of vinegar containing 30 gal. cost $10. How much water must be added that $2.96 may be gained on the whole by selling the mixture @ 36 ct. a gal.?

50. The mean height of six mountains is 10,357 feet. Find the aggregates of their heights. What must be the height of a seventh mountain if the mean height of the seven is 10,643 ft.?

51. In 400 civil years there are 303 years of 365 days each, and 97 years of 366 days each. Find the average length, to the nearest second, of the 400 civil years. In the Persian civil year there are 8 leap years in every 33 years. What is the average length of the Persian civil year?

52. A man bought two cows for $35 each; he sold one of them for $43 and the other for $32. How much did he gain on the first cow? How much did he lose on the second? How much did he gain on the two together? What was his average gain per cow?

53. A man sold two horses, gaining $32 on one of them and losing $15 on the other. How much did he gain on the two together? What was his average gain?

54. A butcher sold three sheep; on the first he gained $1.25, on the second he lost 53 ct., and on the third he gained 60 ct. How much did he gain on the three, and what was his average gain per sheep?

55. A man paid 40 ct. a day for his board. On Monday he earned $2.00, on Tuesday he earned $1.50, on Wednesday he earned $3.30, on Thursday, which was a holiday, he earned nothing. How much did he earn during the four days over and above his board? How much did he thus clear per day, including Thursday?

56. A merchant gained $2336 in his first year of business, $1875 in his second year, $619 in his third year, lost $987 in his fourth year, lost $1178 in his fifth year, gained $293 in his sixth year, and gained $1361 in his seventh year. Find his average gain for the seven years.

57. A merchant bought 5 barrels of pork. Three of them weighed *more* than 200 lb. each by 1 lb. 8 oz., 3 lb. 4 oz., and 5 lb. 12 oz. respectively, and two weighed *less* than 200 lb. by 2 lb. 4 oz. and 3 lb. 4 oz. respectively. What was the total weight, and what the average weight of the 5 barrels?

58. At 6 successive tides the highest point reached by the water was 1 ft. 2 in. below, 9 in. below, 1 ft. 1 in. above, 2 ft. 4 in. above, 1 ft. 3 in. below, and 1 ft. 3 in. above high-water mark respectively. What was the average above high-water mark for these six tides?

IV. SHARING.

EXERCISE XVI.

1. Divide 24 marbles between Henry and Edward so that Henry may have 4 more than Edward.

2. Annie and Jane together have 17 chickens; Annie has 5 more than Jane. How many has each?

3. Robert has 6 pigeons more than Donald; together they have 20 pigeons. How many has each?

4. Divide 76 cents between Willie and Frank, giving Frank 18 ct. more than Willie.

5. A man walked 37 mi. in two days. The second day he walked 7 mi. more than he did the first. How many miles did he walk each day?

6. Divide $5 between two men, giving the first 50 ct. more than the second.

7. A span of horses weighed 2640 lb.; one of them weighed 176 lb. more than the other. How much did each weigh?

8. Divide $10,000 between Smith and Robinson, giving Smith $1000 more than Robinson.

9. Divide $7770 between a college and a hospital, giving $2000 more to the college than to the hospital.

10. A merchant gained $7955 in two years. He gained $1143 more during the second year than during the first. How much did he gain each year?

11. A horse and cutter were worth $276, the horse being worth $150 more than the cutter. How much was each worth?

12. A merchant invested $7945 in dry goods and groceries, the groceries costing $800 more than the dry goods. How much did he invest in each?

13. Two men together earned $19, of which sum one earned $4 more than the other. How much did each earn?

14. Two parcels of tea together weigh 8 lb., one being 1 lb. 4 oz. heavier than the other. How much does each weigh?

15. Two men divided 31 gal. 2 qt. of coal oil between them, one of them taking 4 gal. more than the other. How much did each take?

16. Two men together chopped 27 cords of wood; one of them chopped 7 cords 48 cu. ft. more than the other. How much did each chop?

17. Two boys were 100 yd. apart. They walked straight toward each other, and when they met one had walked 5 yd. more than the other. How far did each walk?

18. Divide 25 ct. among Thomas, Alfred and Edith, giving Edith 4 ct. more than either Thomas or Alfred.

19. Divide 48 apples among Harry, Annie and Jennie, giving Annie and Jennie each 3 more than Harry.

20. Three boys were to share a dollar among them. The first was to get 10 ct. more than the second, and the second was to get 15 ct. more than the third. How much was each to get?

21. Three hogs weighed exactly 320 lb. The first weighed 14 lb. less than the second, and the second weighed 16 lb. less than the third. What was the weight of each?

22. Three horses were sold for $420. The first brought $21 less than the second, but $15 more than the third. What was the price of each?

23. A piece of cloth 44 yd. long was cut into three pieces; the first was 10 yd. shorter than the second, but 2 yd. longer than the third. What was the length of each?

24. Three milk-cans contained altogether 40 qt. of milk; the first contained 3 qt. more than the second, but 4 qt. less than the third. How much did each contain?

25. The total weight of three boxes of honey was 24 lb.; the second weighed 2 lb. 6 oz. more than the first, but only 10 oz. more than the third. Find the weight of each.

26. A farm of 200 A. was to be divided off among two brothers and a sister; the sister was to receive 50 A. less than the elder brother, who was to receive 20 A. more than the younger brother. What was the share of each?

27. The total weight of four crocks of butter was 122 lb.; the first weighed 4 lb. less than the second, but 9 lb. more than the third, which weighed 5 lb. less than the fourth. What was the weight of each?

28. Divide 25 apples between a boy and a girl, giving the girl 3 apples for every 2 given to the boy.

29. Divide 63 ct. between Harry and Willie so that Willie may get 4 ct. for every 3 given to Harry.

30. Divide 24 ct. between John and James, giving John twice as much as James.

31. Divide a dollar between Agnes and Bella so that Agnes may get thrice as much as Bella.

32. Distribute $44 among three men so that the second may get three times and the third 4 times as much as the first.

33. I have cent and five-cent pieces, an equal number of each, amounting to 24 ct. in all. How many pieces of each kind have I?

34. Willie had 75 ct. in five-cent and ten-cent pieces, an equal number of each kind. How many has he of each kind?

35. Edgar has $2.80 in five-cent, ten-cent and twenty-five-cent pieces, an equal number of each. How many has he of each kind?

36. Four presses strike off at the same rate fifty-cent, twenty-five-cent, ten-cent and five-cent pieces, and the total value of the money coined in 9 hours is $9922.50. How many coins does each press strike off per hour?

37. A mixture of green and black teas is made, 3 oz. of green to every 5 oz. of black. How much of each kind will there be in 2 lb. of the mixture?

38. A mixture of three different grades of sugar is made by putting 3 lb. of the first grade and 4 lb. of the second to every 6 lb. of the third. How many pounds of each grade are there in 208 lb. of the mixture?

39. A business in which there are five partners produces $2090 profit; of this profit the senior partner is to receive 5 shares, the second partner 3 shares, and each of the other three partners one share. What sum is the senior partner to receive?

40. An examiner wishes to distribute a total of 100 marks among three questions so that the second question shall get 8 marks and the third 10 marks for every 7 marks given to the first. How many marks must he assign to each question?

41. The sum of $135 was paid as a week's wages to an equal number of men, women and boys. The men received $1.25, the women 75 ct., and the boy 50 ct. each per day. How many were there of each class?

42. The weekly wages at a mill amounted to $583.20. In the mill there were seven times as many women and twice as many men as there were boys. A man's wages were $1.90 per day, a woman's 90 ct. per day, and a boy's 70 ct. per day. How many women were there in the mill?

43. Divide 36 apples among 3 boys and 2 girls so that each girl may receive 3 apples more than each boy.

44. William had $2.05 in twenty-five-cent and ten-cent pieces, there being three more ten-cent pieces than twenty-five-cent pieces. How many ten-cent pieces were there?

45. Jennie has a dollar in five-cent and ten-cent pieces, the number of ten-cent pieces being less by 2 than the number of five-cent pieces. How many five-cent pieces has she?

V. MEASUREMENTS.

A **Rectangle** is a flat figure enclosed by four straight lines and having all its angles equal to one another.

Examples.—A page of a book, the face of a postal card, each of the faces of a common brick.

A **Square** is a rectangle that has all its sides equal.

Examples.—A chess-board and the checks marked on it.

A RECTANGULAR or QUADRATE SOLID or **Quad** is a solid enclosed by six rectangles.

Examples.—A brick, a common packing case, a plank.

A **Cube** is a quad whose six faces are squares.

Examples.—Dice.

The **dimensions of a surface** are its *length* and its *breadth*.

The **dimensions of a solid** are its *length*, its *breadth*, and its *thickness*.

In writing down the dimensions of surfaces and of solids, the sign × is used to denote the word *by*, an accent (′) to denote the word *feet*, and two accents (″) to denote the word *inches*. Thus the dimensions of a rectangle 3 ft. long and 2 ft. wide would be denoted by 3′ × 2′, read "three feet by two feet." If a plank were 12 ft. long, 8 in. wide and 2 in. thick, its dimensions would be denoted by 12′ × 8″ × 2″ read "twelve feet by eight inches by two inches."

EXERCISE.—Measure to the nearest inch and express in accent notation the dimensions of—

1. One face of your slate.
2. A page of this book.
3. A page of your copy-book.
4. The top of your desk.
5. The blackboard.
6. A brick.
7. Any box.
8. A gallon measure.

LINEAR MEASUREMENTS.

The **Perimeter of any surface-figure** is the sum or total length of the lines which bound the figure.

EXERCISE XVII.

Draw rectangles of the following dimensions and *measure* their perimeters:

1. 3″ × 2″. *3.* 3″ × 3″. *5.* 1′ × 6″. *7.* 1′ × 1′. *9.* 2′ 6″ × 1′ 6″.
2. 3″ × 4″. *4.* 5″ × 4″. *6.* 1′ × 1″. *8.* 1′ 3″ × 3″. *10.* 3′ × 3′.

11. The floor of a room is a rectangle 15′ × 12′. What is its perimeter?

12. A rectangular room is 22 ft. long by 14 ft. wide. What is the perimeter of the ceiling?

13. The ceiling of a room is a rectangle 16′ × 13′. What is the length around the walls?

14. A rectangular room whose dimensions are 22′ × 14′ has two doors with frames 3′ 10″ wide each and three windows with frames 4′ 3″ wide each. Find the length round the room less the total width of the door-frames and the window-frames.

15. A rectangular room 28′ × 15′ has two doors with frames 3′ 8″ wide, and six windows with frames 3′ 10″ wide. Find the length round the room less *half* the total width of the door-frames and the window-frames.

16. Find the cost of fencing a rectangular field 40 rd. × 60 rd. at $1.40 per rod.

17. Find the cost of fencing a rectangular building-lot of 4 rd. frontage by 8 rd. in depth at a cost of 45 ct. per yard for the front fence and 15 ct. per yard for the sides and the rear. (2 rd. = 11 yd.)

18. Find the cost of fencing a rectangular corner-lot 66′ × 132′, the street fence costing 55 ct. a yd. and the line fences 25 ct. a yd., but only half of the cost of the latter to be charged to the lot.

19. How many rails 11 ft. long would be required to enclose a rectangular field 30 rd. × 48 rd. with a straight fence 6 rails high?

20. Find the cost of the wire at 6 ct. per 5 yd. for a barbed-wire fence five wires high to enclose a rectangular field 36 rd. × 45 rd.

21. Find the lengths of the perimeters of the several faces of a brick 8″ × 4″ × 2″. Find the total length of the edges of the brick.

Carpeting is made of various widths and is sold by the yard of length. The more common widths are 27 in. and 36 in.

In determining the number of yards of carpeting required for a room, *first decide whether the strips shall run lengthwise of the room or across it, and then find the number of strips needed. The length of a strip multiplied by the number of strips will give the total length of carpeting required.* In determining the length of the strips, *allowance must be made for waste in matching the patterns.*

EXERCISE XVIII.

1. How many strips of carpeting 30 in. wide will be required for a rectangular floor 22′ × 15′, if the strips run lengthwise of the room?

2. How many strips of carpeting 27 in. wide will be required for a rectangular floor 24′ × 13′ 3″, if the strips run lengthwise of the room? What width will have to be turned under?

3. How many strips of carpeting 33 in. wide will be required for a rectangular room 23′ 9″ × 18′ 9″, if the strips run across the room? How many strips will be required if they run lengthwise of the room? How much will need to be turned under in each case?

4. How many yards of carpeting 40 in. wide will be required for a rectangular room 22′ 8″ × 15′ 10″, if the strips run lengthwise of the room and 9 in. per strip be wasted in matching?

5. How many yards of carpeting 27 in. wide will be required for a rectangular room 17′ 6″ × 14′ 5″, if the strips run across the room and 11 in. per strip be wasted in matching?

6. How many yards of carpeting 33 in. wide will be required for a rectangular room 16′ 3″ × 10′, if the strips run lengthwise of the room and 4 in. per strip be wasted in matching? How many yards would be required if the strips ran crosswise of the room and 6 in. per strip were wasted in matching?

7. Which way must the strips of carpeting a yard wide run to carpet with the fewest possible number of yards a rectangular room 20′ 9″ × 17′ 9″, if there be no waste in matching in either case?

8. Find the cost of the carpet for a rectangular room 22′ 8″ × 13′ 4″, if the carpeting be 27 in. wide and cost $1.75 a yard, and 9 in. per strip be wasted in matching, the strips running lengthwise of the room.

9. What will be the cost of the carpeting a yard wide, at \$1.35 per yd., for a rectangular room 25′ 4″ × 14′ 8″, the strips being laid lengthwise of the room and 8 in. per strip being wasted in matching? What would be the cost if the strips were laid across the room and 4 in. per strip were wasted in matching?

10. Find the cost of carpeting a rectangular room 28′ 10″ × 17′ 8″, if the strips, 27 in. wide, run lengthwise of the room and 9 in. per strip be wasted in matching, the carpeting costing \$2.10 a yd. and 10 ct. a yd. for making and laying?

11. How many yards of stair-carpet will be required for a straight stair of 20 steps 11 in. wide, with 7 in. rise, allowing 1 yd. for extra at top?

12. Find the cost of the stair-carpet at \$1.15 a yd. for a flight of stairs of 24 steps 13 in. wide, with 7 in. rise, allowing 1 yd. extra at top and 2 yd. 2 ft. at the turn of the stairs.

Canadian wall-paper is made in rolls 8 yd. long and in double-rolls 16 yd. long, the width in both cases being 21 in.

In determining the number of rolls required to paper a room of ordinary height, *find the number of strips 21 in. wide required to go round the room, leaving out the full width of the doors and the windows; a double-roll, or two single rolls, will be required for every 5 strips. (For U.S. wall-paper, reckon in strips 18 in. wide.)*

EXERCISE XIX.

1. How many rolls of wall-paper will be required for a room 18′ 6″ × 15′ 4″, making deduction for 1 door and 2 windows each 4′ wide and 1 door 3′ 8″ wide?

2. How many rolls of wall-paper will be required for a room of ordinary height 23′ 4″ × 14′ 5″, with 2 doors and 3 windows each 4′ wide?

3. Find the cost of the wall-paper at 75 ct. per roll for a room 21′ 8″ × 13′ 6″, with 2 doors each 3′ 9″ and 3 windows each 4′ 2″ wide.

4. Find the cost of the wall-paper at 45 ct. a roll and bordering at 10 ct. a yard for a room 27′ 9″ × 17′ 3″, allowing for 2 doors each 4′ 2″ wide and 4 windows each 3′ 10″ wide. (The allowance for doors and windows is made on the paper, but not on the bordering.)

5. If a roll give only *two* strips, and 9 strips be deducted for doors and windows, find the cost of papering a room 23′ 6″ × 14′ with paper at 65 ct. per roll and bordering at 7 ct. per yd., hanging the paper costing 15 ct. per roll.

AREAS OF RECTANGLES.

The **Area of any surface-figure** is the measure of the surface enclosed by the lines which bound the figure.

The numerical value of the area expresses how many times some chosen surface-figure, called *the unit of area*, is contained in the measured figure.

The **unit of area** generally selected is A SQUARE WHOSE SIDE IS SOME STATED UNIT OF LENGTH.

Square brackets [ ] enclosing the dimensions of a surface-figure denote that the figure is a rectangle. A number written immediately outside the brackets denotes that number of rectangles of the dimensions noted within the brackets. Thus $[4'' \times 3'']$ denotes a rectangle 4 in. long by 3 in. wide; $[1' \times 1']$ denotes a square 1 ft. long by 1 ft. wide—that is, a square foot; $6\,[3' \times 2']$ denotes 6 rectangles each 3 ft. by 2 ft.; $4 \times 5\,[4'' \times 4'']$ denotes 4 times 5 squares 4 in. by 4 in.—that is, 20 squares each 4 inches square.

EXERCISE XX.

Read the following and draw the figures denoted:—

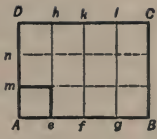
1. $[3'' \times 2'']$. *3.* $[3'' \times 3'']$. *5.* $3\,[2'' \times 2'']$. *7.* $[1'\,3'' \times 7'']$.
2. $[1'' \times 1']$. *4.* $2\,[1'' \times 1'']$. *6.* $4\,[4'' \times 2'']$. *8.* $2\,[1'\,6'' \times 4'']$.

Read—

9. [2 yd. × 1 yd.] *10.* [12 mi. 880 yd. × 99 ft.]

Express the following in bracket notation:—

11. A rectangle 8 in. long by 5 in. wide.
12. A rectangle 1 ft. long by 3 in. wide.
13. Three rectangles 7 in. by 4 in.
14. A rectangle 4 ft. 3 in. long by 2 ft. wide.
15. A rectangle 2 ft. 6 in. long by 1 ft. 9 in. wide.
16. A rectangle 25 yd. long by 5 yd. wide.
17. A rectangle 20 mi. long by 100 ft. wide.
18. A square inch.
19. A square foot.
20. A square yard.
21. A square mile.
22. Six square inches.
23. Six inches square.
24. Three square feet.
25. Three feet square.



Example.—Let the figure $ABCD$ be a rectangle whose length AB is 4 units and breadth AD is 3 units. Mark off AB into 4 parts, Ae, ef, fg, gB, each *one unit long*. Through e, f and g draw the straight lines eh, fk, gl, dividing $ABCD$ into the four rectangles $AehD$, $efkh$, $fglk$, $gBCl$. Each of these rectangles will be 1 unit wide by 3 units long. Hence

$$1\ [4 \text{ units} \times 3 \text{ units}] = 4\ [1 \text{ unit} \times 3 \text{ units.}]$$

Mark off AD into 3 parts Am, mn, nD, each *one unit long*, and through m and n draw straight lines, dividing each of the rectangles $AehD$, $efkh$, $fglk$, $gBCl$ into squares. Each of these rectangles will make 3 squares, and the 4 rectangles together will make 4×3 squares. Hence

$$\begin{aligned} &1\ [4 \text{ units} \times 3 \text{ units}] \\ &= 4\ [1 \text{ unit} \times 3 \text{ units}] \\ &= 4 \times 3\ [1 \text{ unit} \times 1 \text{ unit}] \\ &= 12 \text{ sq. units.} \end{aligned}$$

EXERCISE XXI.

Prove the following statements by drawing the figures and express the final results in Square Measure:—

1. $[3'' \times 2''] = 3\ [1'' \times 2''] = 3 \times 2\ [1'' \times 1'']$.
2. $[5'' \times 3''] = 5\ [1'' \times 3''] = 5 \times 3\ [1'' \times 1'']$.
3. $[6'' \times 4''] = 6\ [1'' \times 4''] = 6 \times 4\ [1'' \times 1'']$.
4. $[2' \times 2'] = 2\ [1' \times 2'] = 2 \times 2\ [1' \times 1']$.
5. $[1' \times 1'] = 12\ [1'' \times 1'] = 12 \times 12\ [1'' \times 1'']$.
6. $[1 \text{ yd.} \times 1 \text{ yd.}] = 3\ [1' \times 1 \text{ yd.}] = 3 \times 3\ [1' \times 1']$.
7. What are the dimensions in inches of a square foot? How many square inches are there in a square foot?
8. What are the dimensions in inches of a square yard? How many square inches are there in a square yard?
9. What are the dimensions in yards of a square mile? How many square yards are there in a square mile?
10. What are the dimensions in inches of 10 ft. square? How many square inches are there in 10 ft. square? How many square inches are there in 10 sq. ft.?

From the preceding examples we may obtain the following rule for determining the area of a rectangle whose dimensions are given :—

Express the length and the breadth of the rectangle in units of the same denomination ; then *the product of the* NUMBER *of units in the length by the* NUMBER *of units in the breadth will be the* NUMBER *of square units of that denomination in the area.*

Hence, also, *if the number of square units in the area of a rectangle be divided by the number of linear units of the same denomination in either side, the quotient will be the number of linear units of the same denomination in the other side.*

EXERCISE XXII.

1. How many square inches are there in a rectangular sheet of paper 17″ × 13″ ?

2. How many square feet are there in the surface of a rectangular table 5′ × 3′?

3. How many square feet are there in the floor of a rectangular room 18′ × 15′ ?

4. Find the area of a blackboard [24′ × 4′].

5. How many square inches are there in the surface of a chess-board 14 inches square ?

6. How many square yards of oilcloth would it take to cover the floor of a rectangular room 21′ × 18′ ?

7. How many square feet of wall would a roll of wall-paper 8 yd. by 21 in. cover, deducting nothing for waste ?

8. How many square yards are there in a village lot [132′ × 66′] ? How many lots of this size would be equal in area to an acre ?

Find the area in acres, etc., of rectangular fields of the following dimensions :—

9. 23 ch. × 10 ch. *11.* 35 ch. square. *13.* 120 yd. square.
10. 15 ch. × 12 ch. *12.* 77 yd. × 33 yd. *14.* 130 ft. × 78 ft.
15. 165 ft. square. *17.* 15 ch. 73 ln. × 12 ch. 25 ln.
16. 12 ch. 50 ln. square. *18.* 13 ch. 75 ln. × 10 ch. 25 ln.

Find the area in sq. yd., etc., of rectangles of the following dimensions :—

19. 12′ 3″ × 9′. *22.* 15′ 7″ × 12′. *25.* 16 yd. × 33″.
20. 18′ × 13′ 6″. *23.* 9′ 9″ square. *26.* 27 yd. × 27″.
21. 24′ 8″ × 15′ 4″. *24.* 3 yd. 2 ft. square. *27.* 18 yd. 1′ 6″ × 40′.

28. How many acres are required for a railway 100 miles long by 99 ft. wide?

29. How many square inches are there in the outside surface of a crayon box [7″ × 4″ × 3″]?

30. Find the number of square inches in the surface of a brick [8″ × 4″ × 2″].

31. How many square feet are there in the outside face of a tight-board fence 6 ft. high round a rectangular lot 132′ × 66′?

32. Find the total area of the *walls* of a room [18′ × 13′ × 10′].

33. Find the total area of the walls and ceiling of a room [16′ 6″ × 12′ 6″ × 10′ 6″].

34. The lid of a box is 6″ wide and its area is 54 sq. in. How long is it?

35. A rectangular room is 18′ long and its floor contains 234 sq. ft. How wide is the room?

36. The top of a table is a rectangle 30″ wide and its area is 10 sq. ft. What is its length?

37. How many yards of carpet 27 in. wide will cover 30 sq. yd.?

38. How many yards of carpet 30 in. wide will cover 40 sq. yd.?

39. The area of the floor of a rectangular room is 246 sq. ft. 96 sq. in. and the width of the room is 13′ 4″. Find the length of the room.

40. A square foot of paper is cut into rectangular pieces 3″ × 2″. How many pieces are there?

41. How many pupils would a rectangular school-room 36′ × 22′ 6″ accommodate, allowing 10 sq. ft. of floor per pupil?

42. Thirteen hundred and fifty men stood on a rectangular space 20 yd. by 10 yd. How many square inches on an average did each man occupy?

43. A lot 99 ft. deep is sold at the rate of $35 per foot of frontage. What rate is that per acre?

The unit or standard of measurement of painting, paving, plastering, ceiling and wainscotting is the square yard. In estimating the amount of any of these kinds of work—

Measure the total area within the boundary lines of the work, including all openings; from this gross area deduct HALF *the area of all doors, windows and other openings, and take as the net area the* WHOLE NUMBER *of square yards* NEAREST *to the remainder.*

Laths are put up in bundles of 100 pieces each 4 ft. long. A bundle is estimated to cover 5 sq. yd. In estimating the number of bundles of laths deduct the *whole* area of all openings.

The unit of measurement of roofing and flooring is a **Square** of 100 sq. ft.

Shingles are estimated to average 4 in. wide, so that a shingle laid 4 in. to the weather should cover 16 sq. in., and 9 shingles should cover 16 sq. in. $\times 9 = 144$ sq. in. $= 1$ sq. ft. At this rate 900 shingles would cover a Square; but to allow for waste and imperfections, *it is usual to reckon 1000 shingles to the Square.* Shingles are put up in bunches of 250 each, so that 4 bunches contain 1000 shingles, and to cover a roof 4 bunches will be required per 100 sq. ft., or one bunch for every 25 sq. ft. Hence to estimate the number of shingles required for any roof—

From the total area of the roof deduct the area of all openings in it and divide the remainder by 25; the whole number nearest to the quotient will be the number of bunches required.

EXERCISE XXIII.

Find the cost at 22 ct. per sq. yd. of plastering as follows:

1. Walls and ceiling of room $27' \times 18' \times 10'$; two doors $7' \times 4'$ and four windows $6' \times 4'$.

2. Walls only of room $16' \times 14'\ 3'' \times 10'$; two doors $7' \times 3'\ 6''$, two windows $6' \times 4'$.

3. Walls and ceiling of room $18' \times 15'\ 6'' \times 10'\ 6''$; two doors $7'\ 6'' \times 3'$, two windows $6' \times 3'\ 10''$, one mantel-piece $5' \times 3'\ 6''$.

4. Walls of room $16' \times 15' \times 9'\ 9''$; one door $7' \times 4'$, and three windows $6'\ 6'' \times 4'$.

5. Ceiling only of a room $22' \times 13'\ 6''$.

6–10. Find the number of bundles of laths required for each of the above-mentioned rooms.

11. Find the cost at 15 ct. a sq. yd. of painting both sides of a close board fence 6′ high around a building-lot [$133' \times 66'$], adding $2.50 for painting the posts and the rails.

12. How much would it cost to paint the walls of a cottage-roofed house $27' \times 24' \times 12'$ at 15 ct. per sq. yd.?

13. How much would it cost at 20 ct. per sq. yd. to paint the walls of a house $29' \times 22'$, with side-walls 15′ high and gable-peaks rising 9′ above the side-walls, counting the two gable-peaks equal to one full wall of equal height?

14. How many squares of shingling are there in a roof [50′ × 20′]?

15. How many squares are there in a roof in the form of two rectangles each 30′ × 16′ 8″?

16. How many bunches of shingles will be required to cover a roof [16′ × 12′ 6″]?

17. How many bunches of shingles will be required for a roof 2 [28′ × 15′]?

18. How many bricks 8″ × 4″ × 2″ laid flatwise will be needed to pave a rectangular courtyard 48′ × 30′?

19. How many bricks 8″ × 4″ × 2″ laid on edge will pave a rectangular yard 40′ × 25′?

20. How many sods [2′ × 18″] will be required to sod a piece of ground [36′ × 27′]?

21. A tinsmith used 1200 sheets of tin to cover a roof, each sheet covering [12″ × 9″]. What was the area of the roof?

22. The floor of a hall is [75′ × 40′]. How many squares of flooring are there in it?

23. How many slates at three to the square foot will be required to cover 17 squares of roof?

24. What will be the cost of ceiling a school-room 37′ 6″ × 24′ at 30 ct. a sq. yd?

25. How much will it cost at $1.10 a sq. yd. to pave a street 87 rd. long by 50 ft. wide?

26. How many panes 16″ × 9″ would there be in a box of glass containing 50 sq. ft.?

27. How many panes 30″ × 15″ would there be in such a box?

28. How many boxes of 50 sq. ft. each would be required to glaze 20 windows, each requiring 6′ 8″ × 3′ of glass?

29. A stone-cutter dressed the tops, fronts and ends of three stone steps each 5′ × 12″ × 6″, and of a fourth step 5′ × 2′ 6″ × 6″. How much did the work cost at 18 ct. per sq. ft.?

30. What would it cost at 25 ct. a sq. yd. to gravel a walk 3′ wide around a grass plot 24 ft. square?

31. How much would it cost at 22 ct. per square yard to paint the walls of a rectangular room 27′ × 13′ 6″ × 11′, deducting two doors 7′ × 4′ and three windows 5′ 10″ × 4′?

32. What would it cost to paint the ceiling of the same room at 30 ct. per sq. yd., reckoning full price for a part of a yard?

33. A certain roof contains 9 squares of shingling. How much would it cost to paint it at 18 ct. a square yard?

VOLUMES OF QUADS.

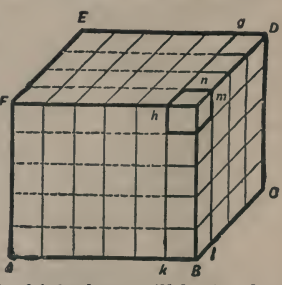
The **Volume of any solid or space-figure** is the measure of the space enclosed by the surfaces which bound the figure.

The numerical value of the volume expresses how many times some chosen volume, called *the unit of volume*, is contained in the measured figure.

The **unit of volume** generally selected is A CUBE WHOSE EDGE IS SOME STATED UNIT OF LENGTH.

Square brackets enclosing the dimensions of a solid denote that the solid is a quad or brick-shaped. A number written immediately outside the brackets denotes that number of quads of the dimensions noted within the brackets. Thus $[6'' \times 4'' \times 3'']$ denotes a quad 6 in. long, 4 in. wide and 3 inches thick; $4[1' \times 1' \times 1']$ denotes 4 cubes 1 ft. long on each edge—that is, 4 cu. ft.

Let the figure *ABCDEF* represent a quad whose length *AB* is 6 units, breadth *BC* is 4 units, and height *CD* is 5 units. Mark off *AB* into 6 parts, *BC* into 4 parts, and *CD* into 5 parts, *each part equal to a unit of length*, and through the points of division draw planes cutting the quad into cubes. Along *AB* there are 6 units, hence there will be 6 slices like *BCDghk*. Along *BC* there are 4 units; hence in the slice *BCDghk* there will be 4 columns like *Blmnhk*, and as this column is 5 units high there will be 5 cubes in it. Hence in the whole quad there will be 6 slices, each containing 4 columns of 5 cubes, or $6 \times 4 \times 5$ cubes $= 120$ cubes in all. Hence



$$
\begin{aligned}
1\ &[6 \text{ units} \times 4 \text{ units} \times 5 \text{ units}] \\
&= 6\ [1 \text{ unit} \times 4 \text{ units} \times 5 \text{ units}] \\
&= 6 \times 4\ [1 \text{ unit} \times 1 \text{ unit} \times 5 \text{ units}] \\
&= 6 \times 5 \times 4\ [1 \text{ unit} \times 1 \text{ unit} \times 1 \text{ unit}] \\
&= 120 \text{ cubic units.}
\end{aligned}
$$

EXERCISE XXIV.

Read the following :—

1. $[1'' \times 1'' \times 1'']$. *3.* $[4'' \times 3'' \times 2'']$. *5.* $3\,[8'' \times 4'' \times 2'']$.
2. $[3'' \times 2'' \times 2'']$. *4.* $5\,[3'' \times 3'' \times 3'']$. *6.* $6\,[2'' \times 6'' \times 2'']$.

Express the following in bracket notation :—

7. A quad 8 in. long, 3 in. wide and 2 in. thick.

8. A quad 4 ft. long, 9 in. wide and 4 in. thick.

9. A quad 16 ft. long by 10 in. wide by 3 in. thick.

10. Five quads 12 ft. long by 6 in. wide by 3 in. thick.

11. 4786 quads 8 in. long, 4 in. wide and 2 in. thick.

12. A cubic inch. *14.* A cubic foot.

13. 24 cubic inches. *15.* 20 cu. yd.

16. A four-inch cube—that is, a cube of which each edge is 4 in. long.

17. Seven 2 ft. cubes.

Prove the following statements by cutting the solids and express the final results in cubic measure :—

18. $[3'' \times 2'' \times 2''] = 3\,[1'' \times 2'' \times 2''[= 3 \times 2\,[1'' \times 1'' \times 2'']$
$= 3 \times 2 \times 2\,[1'' \times 1'' \times 1'']$.

19. $[4'' \times 3'' \times 2''] = 4\,[1'' \times 3'' \times 2''] = 4 \times 3\,[1'' \times 1'' \times 2'']$
$= 4 \times 3 \times 2\,[1'' \times 1'' \times 1'']$.

20. $[6'' \times 4'' \times 3''] = 6\,[1'' \times 4'' \times 3''] = 6 \times 4\,[1'' \times 1'' \times 3'']$
$= 6 \times 4 \times 3\,[1'' \times 1'' \times 1'']$.

21. What are the dimensions in inches of a cubic foot? Express by the bracket notation a cubic foot in inch dimensions. Reduce the cubic foot thus expressed to cubic inches, following the process denoted in problem 20, and explaining each step in the manner of the example on the preceding page.

22. What are the dimensions in feet of a cubic yard? Express by the bracket notation a cubic yard in foot dimensions. Reduce in the manner of last example the cubic yard thus denoted to cubic feet.

23. What are the dimensions in inches of a cubic yard? Express by the bracket notation a cubic yard in inch dimensions. Reduce to cubic inches the cubic yard thus expressed, explaining each step.

24. What are the dimensions in yards of a cubic mile? Express by the bracket notation a cubic mile in yard dimensions. Reduce to cubic yards the cubic mile thus expressed, explaining each step.

From examples such as 18, 19 and 20 of the preceding exercise we may obtain the following rule for determining the volume of a quad whose dimensions are given :—

Express the length, the breadth and the thickness of the quad in units of the same denomination; the continued product of the NUMBER *of units in the length, the* NUMBER *of units in the breadth, and the* NUMBER *of units in the thickness will give the* NUMBER *of cubic units of that denomination in the volume.*

Hence, also, *if the number of cubic units in the volume of a quad be divided by the product of the numbers of linear units in any two dimensions, the quotient will be the number of linear units in the third dimension.*

The unit of measurement of excavations and embankments is the cubic yard. *A cubic yard of earth is called* **a load.**

Hewn timber is generally measured by the cubic foot. Lumber of an inch or more of thickness is measured by **the board-foot,** which is $[1' \times 1' \times 1'']$, 12 board-feet making a cubic foot. Lumber less than an inch thick is reckoned as if it were an inch thick.

Bricklaying is estimated by the thousand bricks, determined either by actual count or else *by reckoning 22 bricks laid in mortar to the cubic foot.* Masonry is generally measured by the cubic yard, but sometimes by the perch. A perch of masonry is not a fixed measure, but differs in different places.

In measuring the materials in walls, deductions must be made for doors, windows and all other openings.

A gallon of pure water weighs 10 lb.

A cubic foot of water weighs 1000 oz. and contains 25 quarts.

A ton of anthracite or hard coal measures 33 cu. ft. A ton of bituminous or soft coal measures 42 cu. ft.

EXERCISE XXV.

1. How many cubic inches are there in a brick $8'' \times 4'' \times 2''$?

2. How many cu. ft. are there in a rectangular box $4' \times 3' \times 2'$?

3. How many cu. ft. are there in a rectangular bin $8' \times 5' \times 4'$?

4. How many cu. ft. are contained in a pile of cordwood $8' \times 4' \times 4'$?

5. How many bricks $8'' \times 4'' \times 2''$ would measure a cubic ft.?

Find the volumes of quadrate solids of the following dimensions:—

6. $6' \times 3' \times 4''$.
7. $2' \times 6'' \times 4''$.
8. $7' \times 3' \times 1'\ 4''$.
9. $4' \times 2'\ 6'' \times 1'\ 6''$.
10. $5' \times 4'\ 4'' \times 3'\ 3''$.
11. $12'\ 3'' \times 3'\ 4'' \times 2'$.

12. Find the cubic contents of a stick of square timber $24' \times 15'' \times 15''$.

13. How many cu. yd. are there in a rectangular excavation for a cellar $24' \times 18' \times 5'$?

14. How many cu. yd. are there in a rectangular embankment $198' \times 12' \times 10'$?

15. How many cords are there in a pile of cordwood 64 ft. long by 6 ft. high?

16. How many cords are there in a rectangular pile of stones 48 ft. by 12 ft. by 3 ft.?

17. How many tons of hard coal will a rectangular bin 9′ long 5′ 6″ wide and 4′ deep hold?

18. How many tons of soft coal can be put in a rectangular bin 7 ft. long, 4 ft. wide and 3 ft. deep?

19. A rectangular cistern is $6' \times 4' \times 4'$. What will be the weight of the water in it when the cistern is full? How many gallons will the cistern hold?

20. A rectangular bin $8' \times 6' \times 4'$ is full of wheat. How many bushels of wheat by measure are there in the bin, and how much would the whole weigh at 61 lb. to the measured bushel?

21. How many bricks will be required to build a wall 124 ft. long, 33 ft. high and 8 in. thick?

22. How many bricks will be required for the walls of a house 40 ft. long, 27 ft. front and 15 ft. high, deducting 2 doors $7'\ 6'' \times 4'$ and 8 windows $5' \times 4'$, the walls to be 8″ thick?

23. How many cubic yards of masonry are there in the foundations of a house $39' \times 27'$, the stonework to be 6′ high by 18″ thick?

24. How many cu. yd. of stone are there in a rectangular pile $15' \times 12' \times 6'$?

25. How many cu. yd. of masonry are there in a break water $[1500' \times 15' \times 18']$?

26. How many cubic feet of air are there in a rectangular room $18' \times 13'\ 4'' \times 10'\ 6''$?

27. How many feet of lumber will be required to plank a rectangular playground $166' \times 66'$ with plank 2 in. thick?

28. How many feet of 3-in. plank will be required for 2 mi. 220 yd. of sidewalk 7′ 6″ wide?

How many feet board-measure, are there in—

29.	1	board	16′	long,	12″	wide,	1″	thick?
30.	3	"	12′	"	10″	"	1″	"
31.	12	"	16′	"	9″	"	1″	"
32.	150	"	16′	"	10″	"	1″	"
33.	720	"	14′	"	8″	"	1″	"
34.	45	"	16′	"	9″	"	2″	"
35.	24	"	16′	"	10″	"	2″	"
36.	40	planks	16′	"	10″	"	3″	"
37.	25	"	16′	"	9″	"	3″	"
38.	24	scantlings	18′	"	4″	"	3″	"
39.	48	"	15′	"	5″	"	3″	"
40.	100	"	18′	"	6″	"	4″	"

41. The grade of a portion of street 264 yd. long by 60 ft. wide is to be lowered 1 ft. 9 in. How many cubic yards of earth will need to be removed?

42. How many loads of earth will be removed in digging a rectangular trench 100 yd. long, 5 ft. wide and 27 in. deep?

43. How much will it cost to excavate a cellar [36′ × 20′ × 6′] @ 35 ct. a cu. yd.?

44. Find the value of a pile of cordwood [32′ × 4′ × 7′] @ $4.75 a cord.

45. Find the total price of 8 piles of cordwood 60′ × 4′ × 7′ @ $4.25 a cord.

46. How many gallons of water will fill a rectangular cistern 7′ 6″ × 5′ 4″ × 6′?

47. How many barrels of water will be required to fill a rectangular cistern 9′ × 8′ × 6′ 5″?

48. What is the weight of a piece of squared timber 27′ × 16″ × 16″ weighing 29 lb. to the cu. ft.?

49. How many bricks 8″ × 4″ × 2″ will make a rectangular pile 10′ × 10′ × 5′?

50. How many cords are there in a rectangular pile of stone 48′ × 14′ × 4′ 6″?

51. At $10 per M for materials and labor, what will be the cost of the walls of a square-roofed brick house 41′ × 25′ × 18′, the walls being 8″ thick, making no deductions for openings and corners?

52. At $12 per M, what will be the cost of 2-inch plank for a 3-foot sidewalk on the street sides of a rectangular corner lot 55′ × 108′ 8″?

53. What will be the cost of making a rectangular cutting 60′ 9″ × 16′ 8″ × 8′ @ 66 ct. a cu. yard.?

54. How many cubic yards of earth must be removed to grade down 2480 sq. yd. of ground 2 ft. 3 in.?

55. How many cu. yd. of gravel will one acre of a gravel bed averaging 12 ft. thick yield?

56. How many cu. yd. of gravel will be required for 3 mi. of road, the gravel to be laid 9 ft. wide and averaging 8 in. deep?

57. What length of road will 2480 sq. yd. of a gravel bed averaging 8 ft. thick grade, the gravel on the road to be laid 11 ft. wide and 9 in. deep?

58. In excavating a rectangular cellar 36 ft. long by 6 ft. deep, 200 cu. yd. of earth were removed. What was the width of the cellar?

59. In excavating a rectangular cellar 24′ 6″ long by 18′ 4″ wide, the contractor removed 103 cu. yd. 26 cu. ft. 504 cu. in. of earth. What was the average depth of the cellar?

60. What length of wall 6 ft. high by 2 ft. thick will 9 cords of stone build?

61. What must be the length of a pile of cordwood 6 ft. high to contain 9 cords?

62. What must be the height of the ceiling of a rectangular room 30 ft. long by 24 ft. wide to contain 8040 cu. ft. of air?

63. What must be the height of a pile of cordwood 72 ft. long to contain 12 cords 72 cu. ft.?

64. The body of a cart is 3′ 9″ long by 3′ 4″ wide, inside measure. To what depth will 25 cu. ft. of earth fill it?

65. What length of 2-in. plank 18 in. wide will contain 48 board feet?

66. What length of 3-in. plank 10″ wide will contain 40 board feet?

67. What is the width of a 2-in. plank 16 ft. long which contains 40 board feet?

68. What must be the length of a piece of squared timber 12″ × 11″ to contain 22 cu. ft.?

69. What must be the length of a rectangular bin 4 ft. wide by 5 ft. 4 in. deep to hold 150 bushels?

70. What must be the width of a rectangular bin 8 ft. long by 4 ft. 6 in. deep to contain 6 T. of hard coal?

71. What must be the depth of a rectangular cistern to hold 350 gallons when filled to within 6 in. of the top, if the horizontal section of the cistern is to be 4′ by 3′ 6″?

72. What must be the length of a rectangular block of granite 2′ 1″ wide by 1′ thick that it may be equal in weight to another rectangular block measuring 7′ 1″ by 2′ 4″ by 1′ 3″?

CHAPTER V.

FACTORS, MEASURES AND MULTIPLES.

I. INTEGRAL FACTORS.

The numbers 1, 2, 3, 4, ... are called **Integers, Integral Numbers** or **Whole Numbers.** They are classified into Even Numbers and Odd Numbers.

An **Even Number** is an integer which is exactly divisible by 2. *Examples.*—6, 10, 18.

An **Odd Number** is an integer which is not exactly divisible by 2. *Examples.*—5, 9, 21.

EXERCISE XXVI.

1. Name all the even numbers less than 20.

2. Add 100 to each of the even numbers you have just named and prove that the resulting sums are all even.

3. Name all the odd numbers between 20 and 40.

4. Add 100 to each of the odd numbers you have just named and prove that the resulting sums are all odd.

5. Write down all the even numbers between 125 and 135.

6. Subtract 99 from each of these and prove that the remainders are all odd.

7. Write down all the odd numbers between 144 and 154.

8. Subtract 99 from each of these and prove that the remainders are all even.

9. Write down all the even numbers between 1001 and 1011, and divide each of them by 2. Which of the quotients are even and which are odd?

10. Divide 1056 by 2, divide the quotient by 2, divide the second quotient by 2, and thus continue dividing till you come to an odd number as quotient. This done, how many divisions by 2 have you made?

The **Integral Factors** of a number are any integers *other than one and the number itself* whose product is equal to the number.

Thus 3 and 5 are integral factors of 15, for $3 \times 5 = 15$. In like manner 6 and 10 are integral factors of 60, for $6 \times 10 = 60$; so also 12 and 5 are integral factors of 60, for $12 \times 5 = 60$; 2, 3, 5 and 11 are integral factors of 330, for $2 \times 3 \times 5 \times 11 = 330$.

If a number is the product of a *pair* of factors, each of these is called the **cofactor** of the other with respect to their product.

Thus in $4 \times 5 = 20$, 4 is the cofactor of 5 and 5 is the cofactor of 4, *with respect to 20*.

EXERCISE XXVII.

Resolve the following numbers each into a pair of integral factors:

1. 6.	*3.* 14.	*5.* 25.	*7.* 55.	*9.* 51.
2. 9.	*4.* 21.	*6.* 39.	*8.* 49.	*10.* 133.

Resolve each of the following numbers into three integral factors:

11. 12.	*13.* 28.	*15.* 50.	*17.* 52.	*19.* 170.
12. 18.	*14.* 70.	*16.* 165.	*18.* 105.	*20.* 1001.

Resolve the following numbers into factor and cofactor, each in as many ways as possible:—

21. 12.	*22.* 18.	*23.* 30.	*24.* 60.	*25.* 420

Write down all the numbers less than 53 of which—

26. 3 is a factor. *29.* 12 is a factor.
27. 5 is a factor. *30.* 15 is a factor.
28. 7 is a factor. *31.* *Both* 4 and 5 are factors.

Find the least and the greatest integral factor of—

32. 24.	*33.* 847.	*34.* 725.	*35.* 1313.	*36.* 3333.

Two integers are **prime to each other** if they have no integral factor in common—that is, if no integral factor of the one is an integral factor of the other.

Thus 21 is prime to 16, for the only integral factors of 21 are 3 and 7, and neither of these is found among the integral factors of 16. But 21 is not prime to 12, for $21 = 3 \times 7$ and $12 = 3 \times 4$; hence 3 is a common factor of 21 and 12.

A **Prime Number** or **Prime** is an integer that has no integral factors; *it is*, therefore, *prime to all integers less than itself*.

A **Composite Number** is an integer that can be resolved into two or more integral factors.

Thus, of the numbers less than ten, 1, 2, 3, 5 and 7 are primes, but 4, 6, 8 and 9 are composite; for $4=2\times2$, $6=2\times3$, $8=2\times4$, and $9=3\times3$.

EXERCISE XXVIII.

Which of the following numbers are prime and which are composite:—

1. 6.	*3.* 11.	*5.* 35.	*7.* 59.	*9.* 91.
2. 5.	*4.* 27.	*6.* 39.	*8.* 63.	*10.* 61.

11. Write down all the prime numbers less than 50.

12. Which of these primes are not found in the common multiplication table extending to 10×10? Why?

13. Write in a column all the composite numbers between 31 and 61, and opposite each number write its smallest integral factor and the cofactor of such integral factor.

14. Which of these composite numbers are not found in the common multiplication table extending to 10×10? Why?

15. Which would not be found in the multiplication table extended to 20×20? Why? *16.* To 30×30? Why?

Which of the following pairs of numbers are prime to each other, and which have a common factor?

17. 24 and 35. *18.* 40 and 66. *19.* 91 and 98. *20.* 231 and 260.

A **Prime Factor** is a factor which is a prime number.

To resolve a composite number into its prime factors—that is, to find the prime numbers, each repeated as often as necessary, whose product is equal to the given number—

Divide the given number by any prime factor.

If the quotient be composite, divide it in like manner, and so continue until a prime quotient is obtained.

The several divisors and the last quotient put into the form of a continued product will express the resolution of the given number into its prime factors.

In trying for a prime factor, *it is best to try the prime numbers* 2, 3, 5, 7, 11, etc., *in order*, beginning with the smallest, being particularly careful to divide by each *as often as possible* before passing on to the next larger.

Example.—Resolve 1260 into its prime factors.

$$\begin{array}{r|l} 2 & 1260 \\ 2 & 630 \\ 3 & 315 \\ 3 & 105 \\ 5 & 35 \\ & 7 \end{array}$$

Therefore $1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$.

EXERCISE XXIX.

Resolve into prime factors :—

1. 15.	*4.* 108.	*7.* 128.	*10.* 555.	*13.* 1089.
2. 12.	*5.* 112.	*8.* 324.	*11.* 539.	*14.* 289.
3. 36.	*6.* 90.	*9.* 252.	*12.* 860.	*15.* 437.

Resolve the following numbers into pairs of integral factors prime to one another in each case :—

16. 24 *17.* 45. *18.* 60. *19.* 144. *20.* 240

21. Find all the integers less than 16 and prime to it.

22. Find all the integers less than 36 and prime to it.

23. Find all the integers less than 48 and prime to it.

24. If the number of integers less than 16 and prime to it be multiplied by the number of integers less than 3 and prime to it, prove that the product will be equal to the number of integers less than 48 ($=16 \times 3$) and prime to it.

25. Prove that if the number of integers less than 9 and prime to it be multiplied by the number of integers less than 5 and prime to it, the product will be equal to the number of integers less than 45 ($=9 \times 5$) and prime to it.

26. Prove that if 6 be added to the product of the first five digits 7 will be a factor of the sum.

27. Prove that if 10 be added to the product of the nine digits 11 will be a factor of the sum.

II. MEASURES.

One number is a **Measure** of another number if it is contained in that other an exact number of times.

Thus 4 is contained 5 times in 20 *without remainder;* therefore 4 is a measure of 20. But 4 is not a measure of 23, for on dividing 23 by 4 there is a remainder, 3. The reason for calling 4 a *measure* of 20 but not a measure of 23 is this:—With a rod 4 ft. long with no division marks upon it, you could measure off a length of 20 ft., but not a length of 23 ft. Similarly, using only a 4-lb weight you could weigh out 20 lb. of a commodity, but not 23 lb.; with a 4-pt measure you could measure out 20 pt. but not 23 pt.; with nothing but four-dollar bills you could count out a sum of $20, but not a sum of $23.

A **Common Measure** of two or more numbers is a number which measures each of them.

Thus 6 is a common measure of 24 and 30, $5 is a common measure of $25 and $40, and 1 ft. is the only integral common measure of the three lengths, 6 ft., 10 ft. and 15 ft.

The **Greatest Common Measure** of two or more numbers is the GREATEST number that measures each of them. The words *Greatest Common Measure* are usually denoted by their initial letters, G. C. M.

Thus *all* the integral or common measures of 36 and 60 are 1, 2, 3, 4, 6 and 12, and of these 12 is the greatest; the G.C.M. of 36 and 60 is therefore 12.

If two numbers have no common measure whatever, they are **Incommensurable** with respect to each other.

Thus 4 ft. and 6 lb. are necessarily incommensurable, as likewise are 5 pt. and 10 min., for they express quantities differing from each other *in kind*; but it can be proved that the lengths of the side and the diagonal of a square, *although both are lengths and therefore quantities of the same kind,* cannot be expressed by numbers commensurable with respect to one another. Nor is the area of a circle commensurable with the area of a square whose side is equal to the diameter of the circle, although both quantities are areas.

To Find the G. C. M. of Two or More Numbers :—

If the given numbers have all the same unit, *i.e.*, are all alike ;

Arrange the given numbers in a horizontal line ;

Resolve one of the given numbers into its prime factors ;

Use these prime factors as successive *trial-factors of the other given numbers, cancelling those trial-factors that do not measure,* in proper succession, every one *of the given numbers.*

The product of the uncancelled factors and the common unit of the given numbers *will be the G. C. M. required.*

If the given numbers are prime to each other, their G. C. M. is their common unit.

If the given numbers are unlike, they must, if possible, be reduced to equivalent like numbers. If such reduction is not possible, the numbers are incommensurable.

EXERCISE XXX.

Write down all the integral measures of—

1. 6. *2.* 12. *3.* 20. *4.* 30. *5.* 84.

6. Write all the integral measures of 24 in one line, all those of 36 in a second line, and in a third line all the measures common to the first two lines. Prove that the third line consists of all the measures of the greatest number in it—that is, of the G. C. M. of 24 and 36?

Form similar tables for—

7. 36 and 48.
8. 45 and 60.
9. 108 and 144.
10. 84, 126 and 210.

Find the G. C. M. of

11. 48 and 50.
12. 60 " 75.
13. 45 " 72.
14. 120 " 150.
15. 210 " 350.
16. 440, 770, and 1210.
17. 560, 1008, " 1232.
18. 980, 1380, " 1960.
19. 1386, 2268, " 3150.
20. 1820, 6370, 8099, and 10101.

The preceding method of finding the G. C. M. of two or more numbers requires one of them to be resolved into its prime factors ; it is, therefore, applicable only when one at

least of the given numbers can be so resolved. If all the given numbers are difficult of resolution into their prime factors, the following method of finding the G. C. M. may be adopted :—

To find the G. C. M. of two like numbers, divide the greater by the less; then the divisor by the remainder, if there be any; then the first remainder by the second remainder, if there be any; and so continue to divide until there is no remainder. The last divisor will be the G. C. M. required.

Should it be required to find the G. C. M. of more than two numbers, find the G. C. M. of two of them; then of this measure and a third of them; then of this second measure and a fourth number; and thus proceed throughout the given numbers. The last measure found will be the G. C. M. required.

EXERCISE XXXI.

Find the G. C. M. of—

1. 111 and 111111.
2. 279 ft. and 217 ft.
3. 836 lb. and 926 lb.
4. 6993 gr. and 8991 gr.
5. 1001 bu. and 2001 bu.
6. 6307 gal. and 10812 gal.
7. 634 yd. and 634 rd.
8. 12341 ft. and 1394 rd.
9. 10011 and 146969.
10. 14003 and 10013.
11. 12341 and 30401.
12. 38683 and 80497.
13. 40693 and 92999.
14. 4934493 and 7381830.
15. 10336608 and 12226272.
16. 68, 102 and 153.
17. 1628, 2882 and 4543.
18. 5040, 7770, 9912 and 10773.
19. \$3.33, \$37 and \$8.51.
20. 1 mi., 1 rd. and 1 yd.

21. A keg holding 100 lb. of water and a cask holding 25 gal.

22. Divide the numbers given in problem 9 above by their G. C. M., and prove that the quotients are prime to each other. Same for problems 10, 11 and 12.

23. In problem 9 above divide all the remainders by the G. C. M. of the given numbers, and prove that there is no common integral factor of all the quotients. Same for problems 10, 11 and 12.

24. In problem 18 divide the given numbers by their G. C. M., and by resolving the resulting quotients into their prime factors prove that there is no common integral factor of all four quotients.

III. MULTIPLES.

If a number be multiplied by an integer, the product is called a **Multiple** of the number.

Thus $\$2 \times 3 = \$6$, therefore $\$6$ is a multiple of $\$2$; 4 in. $\times 5 =$ 20 in., therefore 20 in. is a multiple of 4 in.

If the multiplier be 2, the product is called the *second* multiple; if 3, the *third* multiple; if 4, the *fourth* multiple, etc.; the number itself is called the first or *prime* multiple. If a number and its multiples be arranged in a column, with the multiplying integers in a side column, the result is called a Multiple Table. The Multiplication Table is simply a table of multiples.

Example.	
1,	25
2,	50
3,	75
4,	100
5,	125
6,	150

If a number be a multiple of each of two or more numbers, it is called a **Common Multiple** of these numbers.

Thus 12 is found among the multiples of 2, 3 and 4; it is therefore a *common multiple* of 2, 3 and 4.

The SMALLEST of all the common multiples of two or more numbers is called the **Least Common Multiple** of these numbers.

Thus 12, 24, 36 and 48 are all found in the multiple tables of both 4 and 6; hence these are, all of them, common multiples of 4 and 6. But no number less than 12 is found in both tables; therefore 12 is the *least common multiple* of 4 and 6.

The words "least common multiple" are usually abbreviated into L. C. M.

EXERCISE XXXII.

Form a table of the first nine multiples of—

1. 13. *2.* 14. *3.* 15. *4.* 48. *5.* 245.

6. Form tables of the first 12 multiples of 3, 4 and 6; select the multiples common to the three tables, and form them into a table of common multiples.

Find a common multiple of—

7. 5 and 6. *8.* 6 and 8. *9.* 9 and 12.

10. Of what two integers is 24 a common multiple ?

11. Of what two, 30 ? *12.* 48 ? *13.* 60 ? *14.* 100 ?

15. 8×12 is a common multiple of 8 and 12. Find all the integers of which 8×12 is a multiple, and form a table of those which are common multiples of 8 and 12.

16. What is the L. C. M. of 8 and 12 ?

17. 120 is a common multiple of 10 and 12. Find all the integers of which 120 is a multiple, and form a table of those which are common multiples of 10 and 12.

18. Find from the answer to question 17 the L. C. M. of 10 and 12.

19. Find the L. C. M. of 8 and 10, and form a table of its first five multiples.

20. Resolve 12 and also 21 into factors prime to each other, and find the L. C. M. of all the factors taken as separate numbers. Prove by actual division that the common multiple thus found is a common multiple of 12 and 21.

On comparing the definitions of measure and multiple it is evident that **if one of two numbers be a measure of the other, the second will be a multiple of the first,** and therefore that a multiple of any number is also a multiple of every measure of that number. From this it follows that a common multiple of two or more numbers must contain *every* factor contained in any one of the numbers, but *a factor* occurring in any one of the numbers *need not be repeated on account of occurring in a second number.* It also follows that only commensurable quantities can have a common multiple.

Thus 12 in. is a *measure* of 60 in., and 60 in. is a *multiple* of 12 in., and consequently of every measure of 12 in., namely, of 1 in., 2 in., 3 in., 4 in. and 6 in.

Also, 60 in. is a common multiple of 12 in. and 20 in., hence all the factors of 12 in., and also all those of 20 in., must be factors of 60 in. ; but factors of 12 in. which also occur in 20 in. need not be repeated, on account of occurring in both 12 in. and 20 in.

$$12 \text{ in.} = 1 \text{ in.} \times 2 \times 2 \times 3.$$

$$20 \text{ in.} = 1 \text{ in.} \times 2 \times 2 \times 5.$$

$$60 \text{ in.} = 1 \text{ in.} \times 2 \times 2 \times 3 \times 5.$$

EXERCISE XXXIII.

Find the L. C. M. of—

1. 4, 10, 12. *3.* 4, 8, 16. *5.* 12, 18, 30, 45. *7.* 33, 9, 12, 22.
2. 8. 12, 15. *4.* 10, 12, 16. *6.* 8, 28, 21, 35. *8.* 18, 7, 11, 9.
9. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
10. 36, 45, 88, 120, 54, 99, 60, 108, 72, 66.
11. 24, 42, 35, 52, 36, 63, 273, 112, 126. 156, 18.
12. 69, 147, 115, 154, 210, 207, 693, 385.

SECOND METHOD.—*To find the L. C. M. of two commensurable numbers, divide one of them by their G. C. M. and multiply the other by the quotient; the product will be the L. C. M. required.*

Should there be more than two numbers, find the L. C. M. of two of them; then of this common multiple and another of the numbers; then of this second common multiple and a fourth number; and so continue throughout the given numbers. The last common multiple found will be the L. C. M. required.

If the given numbers are unlike or compound they must be reduced to equivalent like numbers.

EXERCISE XXXIV.

Find the L. C. M. of—

1. 217 and 279.
2. 1921 and 1469.
3. 6993 and 10989.
4. 5724 and 7733.
5. 6061 and 7337.
6. 30401 and 12341.
7. 2318, 3111 and 3553.
8. 5040, 7770, 9912, 10773.

9. What is the least number which, as cofactor of 24, will yield a multiple of 30 as product?

10. What is the least number by which 204 must be multiplied to yield a multiple of 650?

11. The multiplicand is 4095; the product is a multiple of 3906. Find the least multiplier.

12. Find the least multiplier which, with 8645 as multiplicand, will yield as product a number which is a multiple of both 1001 and 1045.

CHAPTER VI.

FRACTIONS.

I. NOTATION AND NUMERATION.

If any quantity be divided into two equal parts, each part is called a half of the quantity; if into three equal parts, a third of it; if into four, a quarter or fourth of it; if into five, a fifth of it; if into six, a sixth of it; and so on. These parts—a half, a third, a quarter, a fifth, etc.—are called **Fractional Parts** of the quantity.

EXERCISE XXXV.

1. Draw a line 4 in. long and divide it into 2 equal parts. What part of the whole line is each of these parts?

2. Subdivide each part into 2 equal parts. Into how many equal parts is the line now divided? What part of the whole line is each of them?

3. Draw a line 3 in. long and divide it into 3 equal parts. What part of the whole line is each of these parts?

4. Subdivide each of the three parts into 2 equal parts. Into how many equal parts is the line now divided? What part of the whole line is each of them?

5. Draw a line 5 in. long and divide it into 5 equal parts. What part of the whole line is each of these parts?

6. Subdivide each of the 5 parts into 2 equal parts. Into how many equal parts is the line now divided? What part of the whole line is *one* of these parts? 3 of them? 7 of them?

7. Draw a line 6 in. long and divide it into 3 equal parts. What part of the whole line is each of these parts?

8. Subdivide the 3 parts, each into 4 equal parts. Into how many equal parts is the line now divided? What part of the whole line is one of them? 5 of them? 11 of them?

9. How many halves of a slate-pencil are equal to the whole of it?

10. How many thirds? *12.* How many tenths?

11. How many quarters? *13.* How many twelfths?

14. Take a string the length of your slate and double it at the middle; double it again, and yet a third time. What part of the length of your slate is the length of the thrice-folded string? What part would three folds of the string be were they unfolded?

15. If a string be cut into 9 equal parts, what part of the whole string is *one* of the 9 parts? 2 of them? 4 of them? 7 of them?

What is the name of one of the parts of any quantity divided into—

16. 4 equal parts? *18.* 10 equal parts?

17. 5 equal parts? *19.* 19 equal parts?

20. How many halves of anything are equal to the whole of it?

21. How many quarters? *23.* How many tenths?

22. How many eighths? *24.* How many hundredths?

When anything is divided into 12 equal parts what is the name of—

25. Three parts? *27.* Four parts? *29.* Five parts?

26. One part? *28.* Ten parts? *30.* Twelve parts?

What is meant by—

31. Three-quarters of an apple?

32. Two-thirds of a slate-pencil?

33. Three-eighths of a yard?

34. Seven-tenths of a dollar?

35. In 2 apples how many halves of an apple are there? How many quarters? How many thirds?

36. Three oranges would yield how many quarters of an orange? How many eighths?

37. How many tenths of a dollar would be equal to $5.

38. How many twelfths of a foot are there in 7 ft. What is the common name for the twelfth of a foot?

39. Which is larger, one-half of an apple or one-third of it? Why?

40. Which is longer, a third of a yard or a quarter of a yard? Why?

41. Which is heavier, an eighth of a pound or a sixteenth of a pound? Why?

42. Which is the most, three-fifths of a bushel of wheat or three-quarters of a bushel of wheat? Why?

43. Two boys carry a pailful of water, each boy carrying the same weight as the other. What part of the weight does each boy carry?

44. A man spent 3 days on a certain journey, travelling exactly equal distances each day. What part of the journey did he travel per day?

45. A man and a boy did a certain piece of work, the man doing four times as much as the boy. What part of the work did each do?

46. An apple is cut into thirds and one of the thirds is given away. How many are left?

47. If I exchange two dollars for quarters of a dollar, how many quarters should I receive? I pay out three of these quarters. How many have I left?

48. A man owns two-fifths of an acre of land. How much land must he buy to have a whole acre?

49. A man worked five-sevenths of a week, and was idle the rest of the week. What part of the week was he idle? What is one-seventh of a week called?

50. There is just room for two boys to sit on one-third of a certain bench. How many boys of the same size would the whole bench seat?

A unit is any standard used in counting or in measuring.

In 3-quarters of a pound the unit is "a quarter of a pound." But "a quarter of a pound" is, as its name declares, a fractional part of the unit "a pound." Hence the unit in 3-quarters of a pound is itself named as the fractional part of another unit.

In 7-eighths of an inch the unit is "an eighth of an inch," but this, as its name declares, is a fractional part of the unit "an inch." Hence the unit in 7-eighths of an inch is named as the fractional part of another unit.

A unit which is named as a fractional part of another unit is called a **Fractional Unit,** *and the unit of which it is a part is called its* **Prime Unit.**

Thus in the preceding example a quarter of a pound and an eighth of an inch are the *fractional units;* a pound and an inch are the *prime units.*

EXERCISE XXXVI.

What is the Fractional Unit and what its Prime Unit in

1. 3-quarters of an oz.?
2. 4-fifths of an in.?
3. 7-eighths of a bu.?
4. 3-elevenths of a rd.?
5. 2-thirds of a yd.?
6. 5-eighths of a cupful?
7. 4-ninths of a load?
8. 7-twelfths?
9. Half an hour?
10. 9-quarters of a year?

11. How many fractional units are there in each of the numbers in the preceding ten questions?

12. How many of the fractional units of the numbers in each of these ten questions would be required to make one of the corresponding prime units?

13. What is the prime unit and what the fractional unit in Question 14, Exercise XXXVI.

A Fractional Number or Fraction is a Number whose unit is fractional. *A Fractional Number*, therefore, *expresses one or more equal parts of some prime unit.*

To completely express a fraction both the number and the size of the fractional units must be stated. Hence to express a fraction in numerals requires two numbers—one called the **Numerator**, the other the **Denominator.**

The **Numerator** *(that is,* THE NUMBER-TELLER) expresses the number of fractional units in the fraction.

The **Denominator** *(that is,* THE NAME-GIVER) denotes the size of the fractional units by expressing how many of them are contained in the prime unit.

The Numerator and Denominator together are called the **Terms** of the fraction. They are written, the Numerator a little above the Denominator, with a short line between them, so that a fraction is written $\frac{\text{Numerator.}}{\text{Denominator.}}$

Thus the fraction *five-eighths*, which has five for its numerator and eight for its denominator, is written $\frac{5}{8}$.

A fraction expressed in figures is read by first reading its numerator, and then its denominator with the termination of the

corresponding ordinal number, except in the case of fractions with 2 or 4 as denominator, which are read as halves or quarters, as the case may be.

Ex. 1.—$\frac{3}{4}$ is read three-quarters. The 4 expresses that the prime unit—here simply the abstract number 1—is divided into *four* equal parts or quarters; it thus denotes the size of the parts. The 3 expresses that the fraction consists of *three* of these quarters.

Ex. 2.—$\frac{5}{12}$ ft. is read five-twelfths of a foot. The 12 expresses that the prime unit, a foot, is divided into *twelve* equal parts or twelfths; it therefore denotes the length of the fractional unit. The 5 expresses that the fraction consists of *five* of these twelfths.

EXERCISE XXXVII.

Read and analyze in the manner of Examples 1 and 2 above—

1. $\frac{1}{2}$ in.	*4.* $\frac{3}{10}$ gal.	*7.* $\frac{13}{40}$ A.	*10.* $\frac{19}{12}$ sq. ft.
2. $\frac{3}{4}$ yd.	*5.* $\frac{17}{25}$ cwt.	*8.* $\$\frac{47}{100}$.	*11.* $\frac{7}{10}$.
3. $\frac{5}{8}$ lb.	*6.* $\frac{11}{32}$ cord.	*9.* $\frac{8}{9}$ cu. yd.	*12.* $\frac{8}{21}$.

Write in numerals—

13. Five-eighths of a lb.	*17.* Thirteen thirty-seconds.
14. Half an oz.	*18.* Ninety-one hundredths.
15. Three-quarters of a cord.	*19.* Eighteen quarter-hours.
16. Eleven-twelfths of a yd.	*20.* Seven-sixtieths of an hr.

Express—

21. $\frac{3}{4}$ yd. in inches.	*27.* $\frac{8}{9}$ cu. yd. in cu. ft.
22. $\frac{5}{8}$ lb. in oz.	*28.* $\frac{13}{12}$ sq. ft. in sq. in.
23. $\frac{17}{25}$ cwt. in lb.	*29.* $\frac{3}{4}$ cord in cord ft.
24. $\frac{11}{32}$ cord in cu. ft.	*30.* $\frac{11}{12}$ yd. in ft. and in.
25. $\frac{13}{40}$ A. in sq. yd.	*31.* 18 quarter-hr. in hr. and min.
26. $\$\frac{47}{100}$ in ct.	*32.* $\frac{7}{60}$ hr. in min.

33. Show that $\frac{3}{10}$ gal. is a pint and a half.

34. A boy has to walk one mile. When he has walked $\frac{2}{5}$ of the mile, how many yards has he still to walk?

35. A girl has to knit 900 stitches. When she has done $\frac{13}{18}$ of her task, how many stitches will she still have to knit?

II. REDUCTION OF FRACTIONS.

On the basis of value fractions are divided into **Proper Fractions** and **Improper Fractions.**

A Proper Fraction is a fraction whose value is less than 1 or than a Prime Unit. Its numerator must therefore be less than its denominator, for the fraction must not contain as many of the fractional units or parts as its Prime Unit does.

Examples.—$\frac{3}{4}$, $\frac{4}{10}$ in., $\frac{11}{16}$ lb., are Proper Fractions.

An Improper Fraction is a fraction whose value is not less than 1 or than a Prime Unit. Its numerator must therefore be equal to its denominator or greater than it, for the fraction must contain *at least* as many of the fractional units or parts as its Prime Unit does. An Improper Fraction is therefore equal either to an integer or to a number consisting of an integer *and* a fraction.

Examples.—$\frac{5}{4}$, $\frac{10}{10}$ in., $\frac{24}{16}$ lb., are Improper Fractions.

A number consisting of an integer and a fraction is called a **Mixed Number.**

Examples.—$1\frac{1}{2}$, $4\frac{3}{4}$ yd., $6\frac{3}{10}$ hr., are Mixed Numbers. These are read: One and a half, four yards and three-quarters, six hours and three-tenths.

REDUCTION OF WHOLE NUMBERS AND OF MIXED NUMBERS TO EQUIVALENT IMPROPER FRACTIONS.

EXERCISE XXXVIII.

1. How many halves of an apple are there in 3 apples? How many in 4 apples? In 6 apples? In 10 apples?

2. How many quarters of a lb. are there in 3 lb.? In 5 lb.? In 12 lb.?

3. How many thirds of an in. are there in 2 in.? In 8 in.? In 12 in.?

4. How many halves of an apple are there in $2\frac{1}{2}$ apples? In $3\frac{1}{2}$ apples? In $5\frac{1}{2}$ apples?

5. How many quarters of a dollar are there in $\$2\frac{1}{4}$? In $\$5\frac{3}{4}$?

6. How many eighths of an inch are there in $3\frac{3}{8}$ in.? In $7\frac{3}{8}$ in.?

Example.—Reduce $17\frac{5}{8}$ to eighths.

Calculation.	*Explanation.*
$17\frac{5}{8}$	1 = 8-eighths,
8	therefore 17 = 17 (8-eighths)
141 eighths $= 1\frac{4}{8}1$.	= 136-eighths,
	therefore $17\frac{5}{8}$ = 136-eighths and 5-eighths.
	= 141-eighths $= 1\frac{4}{8}1$.

To reduce a whole number to an equivalent improper fraction with given denominator,

Multiply the whole number by the given denominator; write the product as numerator of the required fraction, and under it write the given denominator.

To reduce a mixed number to an equivalent improper fraction with denominator the same as the denominator of the fractional part of the mixed number,

Multiply the integral part of the mixed number by the denominator of the fractional part, and add the numerator to the product; write the sum as numerator of the required improper fraction, and under it write the denominator.

EXERCISE XXXIX.

Reduce to improper fractions—

1. $2\frac{1}{2}$.	*6.* $15\frac{1}{15}$.	*11.* $303\frac{4}{100}$.	*16.* $909\frac{1}{11}$.
2. $3\frac{1}{3}$.	*7.* $36\frac{1}{20}$.	*12.* $343\frac{3}{100}$.	*17.* $45\frac{4}{40}$.
3. $5\frac{3}{8}$.	*8.* $49\frac{8}{10}$.	*13.* $303\frac{3}{1000}$.	*18.* $400\frac{3}{800}$.
4. $6\frac{4}{11}$.	*9.* $99\frac{9}{10}$.	*14.* $333\frac{3}{300}$.	*19.* $760\frac{32}{700}$.
5. $10\frac{7}{13}$.	*10.* $99\frac{1}{11}$.	*15.* $303\frac{3}{500}$.	*20.* $706\frac{32}{700}$.

21. I have 3 apples. To how many children can I give $\frac{1}{2}$ apple each?

22. I have a string $3\frac{3}{4}$ yd. long. If I cut it into lengths of $\frac{1}{4}$ yd. each, how many pieces shall I have?

23. How many more *pieces* will there be in 12 apples divided into thirds than in 8 apples divided into quarters?

24. Out of \$5 I give a quarter of a dollar each to 17 boys. How many quarters have I left?

25. Which is greater, 5 or $\frac{25}{6}$. By how much is it greater?

REDUCTION OF IMPROPER FRACTIONS TO MIXED NUMBERS.

Example.—Reduce $\frac{13}{4}$ in. to inches.

13 = 3 (4) and 1 over,

therefore 13 qu. in. = 3 (4 qu. in.) and 1 qu. in. over

= 3 in. and $\frac{1}{4}$ in. = $3\frac{1}{4}$ in.

EXERCISE XL.

1. How many whole inches are there in 5 halves of an inch? In 8 halves? In 11 halves? In 21 halves?

2. How many quarts are there in $\frac{9}{4}$ qt.? In $\frac{18}{4}$ qt.? In $\frac{13}{4}$ qt.? In $\frac{23}{4}$ qt.?

3. How many whole yards are there in 12 thirds of a yard? In $\frac{10}{3}$ yd.? In $\frac{16}{3}$ yd.? What is the common name for $\frac{1}{3}$ yd.? Answer the preceding questions, substituting this common name for "third of a yard."

4. How many feet are there in $\frac{7}{2}$ ft.? In $\frac{8}{3}$ ft.? In $\frac{15}{8}$ ft.? In $\frac{13}{12}$ ft.?

5. How many pounds are there in $\frac{11}{4}$ lb.? In $\frac{47}{8}$ lb.? In $\frac{15}{3}$ lb.?

6. How many dollars are there in $\$\frac{18}{4}$? In $\$\frac{37}{10}$? In $\$\frac{14}{5}$? In $\$\frac{93}{20}$? What coin is $\$\frac{1}{4}$? $\$\frac{1}{10}$? $\$\frac{1}{5}$? $\$\frac{1}{20}$?

How many wholes in—

7. 17-halves?

8. 29-quarters?

9. 13-thirds?

10. 23-eighths?

To reduce an improper fraction to an equivalent mixed number,

Diride the numerator by the denominator; the quotient will be the integral part of the mixed number; the remainder will be the numerator, and the denominator of the given fraction will be the denominator of its fractional part.

Should there be no remainder, the quotient will be the whole number equivalent to the given improper fraction.

Example.—Reduce $\frac{77}{9}$ to a mixed number.

Calculation.	*Explanation.*
9)77	$\frac{77}{9}$ = 77-ninths
$8\frac{5}{9} = \frac{77}{9}$.	= 8 (9-ninths) and 5-ninths.
	= 8 and 5-ninths = $8\frac{5}{9}$.

EXERCISE XLI.

Reduce to whole or to mixed numbers—

1. $\frac{19}{4}$ in.	*6.* $\frac{1001}{23}$.	*11.* $\frac{404}{10}$.	*16.* $\frac{9487}{47}$.
2. $\frac{27}{8}$ gal.	*7.* $\frac{3003}{19}$.	*12.* $\frac{404}{100}$.	*17.* $\frac{9487}{470}$.
3. $\frac{19}{7}$ lb.	*8.* $\frac{197568}{144}$.	*13.* $\frac{404}{404}$.	*18.* $\frac{9487}{4700}$.
4. $\frac{184}{15}$ bu.	*9.* $\frac{4768}{96}$.	*14.* $\frac{768}{19}$.	*19.* $\frac{7000000}{100}$.
5. $\frac{143}{13}$ ft.	*10.* $\frac{375383}{365}$.	*15.* $\frac{1973}{23}$.	*20.* $\frac{493684}{100000}$.

21. How many bushels are there in 729 baskets of plums, each basket containing $\frac{1}{8}$ bu.?

22. A number of cakes were cut, each into 6 equal-sized pieces. How many whole cakes could be made out of 25 pieces?

23. A merchant has 78 packages of baking-soda, each containing $\frac{1}{4}$ lb. What is the weight of the whole?

24. Jones, in walking, takes 7 steps to the rod. What part of a rod is one of his steps? How far will he walk in 3000 steps? What is the least number of additional steps he must take in order to have walked altogether an exact number of rods? If he take that number of steps in addition to the 3000, how many rods will he have walked altogether?

25. How many gallons will 12 doz. bottles hold if each bottle hold $\frac{1}{5}$ gal.?

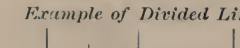
INTERCONVERSION OF DENOMINATORS.

EXERCISE XLII.

1. Draw a line 1 in. long and divide it into 2 equal parts. What is each part called? Subdivide each part into 2 equal parts. Into how many equal parts is the line now divided? What is each of these parts called? Show from the divided line that

$$\frac{1}{2}\text{ in.} = \frac{1\times 2}{2\times 2}\text{ in.} = \frac{2}{4}\text{ in.}$$

Example of Divided Line.



2. Draw a line 3 in. long and divide it into 4 equal parts. Sub-divide each part into 3 equal parts. Show from the divided line that

$$\frac{1}{4} = \frac{1\times 3}{4\times 3} = \frac{3}{12};\quad \frac{2}{4} = \frac{2\times 3}{4\times 3} = \frac{6}{12};\quad \frac{3}{4} = \frac{3\times 3}{4\times 3} = \frac{9}{12}.$$

3. Draw a line 5 in. long and divide it into 5 equal parts. Sub-divide each part into 2 equal parts and show from the divided line that

$$\frac{1}{5}=\frac{1\times2}{5\times2}=\frac{2}{10};\quad \frac{2}{5}=\frac{2\times2}{5\times2}=\frac{4}{10};\quad \frac{3}{5}=\frac{3\times2}{5\times2}=\frac{6}{10};\quad \frac{4}{5}=\frac{4\times2}{5\times2}=\frac{8}{10}.$$

4. Show by cutting an apple or other object that

$$\frac{1}{3}=\frac{1\times2}{3\times2}=\frac{2}{6};\quad \frac{2}{3}=\frac{2\times2}{3\times2}=\frac{4}{6}.$$

5. Show by folding a strip of paper that

$$\frac{1}{3}=\frac{1\times4}{3\times4}=\frac{4}{12};\quad \frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}.$$

$\frac{1}{4}$ is equal to how many—

6. Eighths? *7.* Twelfths? *8.* Sixteenths? *9.* Twenty-fourths?

How many twentieths are equal to—

10. $\frac{1}{2}$? *11.* $\frac{1}{4}$? *12.* $\frac{1}{5}$? *13.* $\frac{1}{10}$?

How many twenty-fourths are equal to—

14. $\frac{1}{2}$? *19.* $\frac{3}{3}$? *24.* $\frac{3}{4}$? *29.* $\frac{2}{6}$? *34.* $\frac{5}{8}$?
15. $\frac{2}{2}$? *20.* $\frac{4}{3}$? *25.* $\frac{4}{4}$? *30.* $\frac{5}{6}$? *35.* $\frac{1\frac{3}{8}}{}$?
16. $\frac{3}{2}$? *21.* $\frac{5}{3}$? *26.* $\frac{5}{4}$? *31.* $\frac{7}{6}$? *36.* $\frac{1}{12}$?
17. $\frac{1}{3}$? *22.* $\frac{1}{4}$? *27.* $\frac{6}{4}$? *32.* $\frac{1}{8}$? *37.* $\frac{5}{12}$?
18. $\frac{2}{3}$? *23.* $\frac{2}{4}$? *28.* $\frac{1}{6}$? *33.* $\frac{3}{8}$? *38.* $\frac{1\frac{5}{12}}{}$?

39. In $\frac{3}{4}$ how many fractional parts are there? How many of these fractional parts are there in one whole? In $\frac{3\times4}{4\times4}$ or $\frac{12}{16}$ how many fractional parts are there? How many of these fractional parts are there in one whole? What is the effect on the *number* of fractional parts of changing $\frac{3}{4}$ into $\frac{3\times4}{4\times4}$? What is the effect on their *size?*

40. If $\frac{5}{8}$ be changed into $\frac{5\times6}{8\times6}$, what has been done to the *five* fractional parts making up the $\frac{5}{8}$? How many of the new fractional parts would make up one whole?

From the problems in the preceding exercise we see that

Multiplying the numerator of a fraction multiplies the *number* of parts making up the fraction; while multiplying the denominator subdivides the parts, for it multiplies the number of them required to make one whole.

Hence *multiplying both terms of a fraction by 2, or 3, or 4, or any other number, does not change the value of the fraction, but is merely equivalent to subdividing each part making up the fraction into 2, or 3, or 4, or other number of equal parts.*

Example.—Reduce $\frac{3}{5}$ to an equivalent fraction, with 35 as denominator.

To do so find the cofactor whose product with 5 is 35, and multiply both the numerator and the denominator by it.

$$5)35 \qquad \frac{3}{5}=\frac{3\times 7}{5\times 7}=\frac{21}{35}.$$

EXERCISE XLIII.

Insert the numerators in—

1. $\frac{3}{4}=\frac{\quad}{16}$.
2. $\frac{4}{5}=\frac{\quad}{20}$.
3. $\frac{3}{8}=\frac{\quad}{48}$.
4. $\frac{4}{15}=\frac{\quad}{60}$.
5. $\frac{7}{18}=\frac{\quad}{72}$.
6. $\frac{10}{21}=\frac{\quad}{441}$.

Insert the denominators in—

7. $\frac{4}{9}=\frac{\quad}{72}$.
8. $\frac{7}{15}=\frac{\quad}{630}$.
9. $\frac{16}{11}=\frac{\quad}{256}$.
10. $\frac{24}{15}=\frac{\quad}{720}$.
11. $\frac{63}{83}=\frac{\quad}{1071}$.
12. $\frac{63}{85}=\frac{\quad}{1701}$.

EXERCISE XLIV.

1. Draw a line 1 in. long and divide it into 4 equal parts. What is each part called? Group these parts into sets of 2 parts each. How many such groups are there in the line. What part of the line is each group? Show from the divided line that

$$\frac{2}{4}\text{ in.}=\frac{2\div 2}{4\div 2}\text{ in.}=\frac{1}{2}\text{ in.}$$

2. Draw a line 3 in. long and divide it into 12 equal parts. Group these parts into sets of 3 each. Show from the divided line that

$$\frac{3}{12}=\frac{3\div 3}{12\div 3}=\frac{1}{4}; \qquad \frac{6}{12}=\frac{6\div 3}{12\div 3}=\frac{2}{4}; \qquad \frac{9}{12}=\frac{9\div 3}{12\div 3}=\frac{3}{4}.$$

When you have finished Turn to The Back of the Book.

3. Show by cutting an apple or other object that

$$\frac{2}{6}=\frac{2\div2}{6\div2}=\frac{1}{3}; \qquad \frac{4}{6}=\frac{4\div2}{6\div2}=\frac{2}{3}.$$

4. Show by folding a strip of paper that

$$\frac{4}{12}=\frac{4\div4}{12\div4}=\frac{1}{3}; \qquad \frac{8}{12}=\frac{8\div4}{12\div4}=\frac{2}{3}.$$

5. Show by grouping the five-cent pieces in a dollar that

$$\frac{15}{20}=\frac{15\div5}{20\div5}=\frac{3}{4}; \qquad \frac{16}{20}=\frac{16\div4}{20\div4}=\frac{4}{5}; \qquad \frac{10}{20}=\frac{10\div10}{20\div10}=\frac{1}{2}.$$

$\frac{2}{3}\frac{4}{6}$ is equal to how many—

6. Twelfths? 7. Ninths? 8. Sixths? 9. Thirds?

How many twelfths are equal to—

10. $\frac{1}{3}\frac{0}{4}$? 11. $\frac{2}{3}\frac{1}{6}$? 12. $\frac{2}{6}\frac{5}{6}$? 13. $\frac{4}{4}\frac{4}{8}$? 14. $\frac{7}{7}\frac{8}{2}$?

Reduce—

15. $\frac{7}{1}\frac{}{4}$ to halves.
16. $\frac{8}{1}\frac{}{2}$ to thirds.
17. $\frac{1}{2}\frac{6}{8}$ to sevenths.
18. $\frac{1}{4}\frac{8}{8}$ to eighths.

19. In $\frac{15}{20}$ how many fractional parts are there? How many of these parts would make up one whole? In $\frac{15\div5}{20\div5}$ or $\frac{3}{4}$ how many fractional parts are there? How many of these fractional parts would make up one whole? What is the effect on the *number* of fractional parts of changing $\frac{15}{20}$ into $\frac{15\div5}{20\div5}$? What is the effect on their *size*?

20. If $\frac{14}{35}$ be changed into $\frac{14\div7}{35\div7}$, what has been done to the *fourteen* fractional parts making up the $\frac{14}{35}$? How many of the new fractional parts would make up one whole?

From the problems in the preceding exercise we see that

Dividing the numerator of a fraction divides the *number* of parts making up the fraction; while dividing the denominator groups the parts, for it divides the number of them required to make up one whole.

Hence *dividing both terms of a fraction by 2 or 3, or 4, or any other number, does not change the value of the fraction, but is merely equivalent to grouping the parts making up the fraction into sets of 2, or 3, or 4, or other number, each as the case may be.*

Example.—Reduce $\frac{42}{48}$ to an equivalent fraction, with 8 as denominator.

To do so find the divisor which, with 48 as dividend, will give 8 as quotient.

$$8)48 \qquad \frac{42}{48} = \frac{42 \div 6}{48 \div 6} = \frac{7}{8}$$

$$6$$

EXERCISE XLV.

Insert the numerators in—

1. $\frac{12}{18} = \frac{\quad}{3}$.
2. $\frac{24}{32} = \frac{\quad}{4}$.
3. $\frac{33}{55} = \frac{\quad}{5}$.
4. $\frac{209}{616} = \frac{\quad}{56}$.
5. $\frac{240754}{200041} = \frac{\quad}{27}$.
6. $\frac{8129291}{1200001} = \frac{\quad}{31}$.

Insert the denominators in—

7. $\frac{72}{126} = \frac{4}{\quad}$.
8. $\frac{847}{1001} = \frac{11}{\quad}$.
9. $\frac{112}{280} = \frac{\quad}{7}$.
10. $\frac{132}{385} = \frac{12}{\quad}$.
11. $\frac{3108}{9990} = \frac{14}{\quad}$.
12. $\frac{4191}{8001} = \frac{11}{\quad}$.

The problems in the last four exercises are examples of the FUNDAMENTAL PRINCIPLE OF FRACTIONS, namely:—

THE VALUE OF A FRACTION IS NOT CHANGED IF ITS TERMS BE BOTH MULTIPLIED OR BOTH DIVIDED BY THE SAME NUMBER.

A fraction is reduced to *lower terms* if a common factor be divided out of both numerator and denominator.

A fraction is expressed in **Lowest Terms** if its terms are integral and prime to each other. Hence

To reduce a fraction to its lowest terms,

Divide both terms by their G. C. M.

EXERCISE XLVI.

Reduce to equivalent fractions expressed in lowest terms—

1. $\frac{18}{27}$.
2. $\frac{15}{25}$.
3. $\frac{21}{49}$.
4. $\frac{48}{256}$.
5. $\frac{84}{51}$.
6. $\frac{39}{143}$.
7. $\frac{118}{188}$.
8. $\frac{625}{1165}$.
9. $\frac{32764}{46114}$.
10. $\frac{13770}{10772}$.
11. $\frac{192843}{12143}$.
12. $\frac{50225}{12500}$.
13. $\frac{15738}{10586}$.
14. $\frac{8000}{10000}$.
15. $\frac{3300}{10092}$.
16. $\frac{9999}{99999}$.
17. $\frac{407}{999}$.
18. $\frac{80016}{109856}$.
19. $\frac{700009}{2832001}$.
20. $\frac{830091}{3709}$.

REDUCTION TO COMMON DENOMINATORS.

If two or more fractions have the same denominator, it is called the *Common Denominator* of the fractions.

Thus $\frac{2}{5}$, $\frac{3}{5}$ and $\frac{7}{5}$ have a common denominator, 5.

To reduce fractions having different denominators to equivalent fractions having a common denominator, a denominator must be found which is a multiple of each of the denominators of the given fractions. This denominator will therefore be a common multiple of the given denominators. If the L. C. M. of them be taken, and it is generally the best to take, the given fractions will be reduced to equivalent fractions with **Least Common Denominator**, provided the given fractions were expressed in their lowest terms or reduced to them before using. Hence

To reduce fractions with different denominators to equivalent fractions with least common denominator,

Reduce the given fractions to lowest terms; find the L. C. M. of the denominators; divide it by the denominator of the first fraction, and multiply both terms of this fraction by the quotient; do likewise with all the other fractions.

Example.—Reduce $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{7}{12}$ to equivalent fractions with least common denominator.

L. C. M. of denominators 4, 8 and 12 is 24.

$24 \div 4 = 6 \qquad \frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$;

$24 \div 8 = 3 \qquad \frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$;

$24 \div 12 = 2 \qquad \frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$.

To determine which of two fractions is the greater they must be reduced to the same fractional unit, and the fraction containing the greater number of such units will be the greater. Reduction to the same fractional unit is effected by reducing the given fractions to a common denominator.

EXERCISE XLVII.

By what numbers must the terms be multiplied to reduce—

1. $\frac{1}{2}$ to 6ths?
2. $\frac{2}{3}$ to 9ths?
3. $\frac{5}{8}$ to 48ths?
4. $\frac{4}{5}$ and $\frac{5}{12}$ to 60ths?
5. $\frac{5}{8}$, $\frac{7}{12}$ and $\frac{3}{20}$ to 120ths?
6. $\frac{5}{7}$, $\frac{7}{9}$, $\frac{9}{11}$, $\frac{11}{13}$, $\frac{16}{21}$ and $\frac{21}{26}$ to 9009ths?

What is the least common denominator to which can be reduced—

7. $\frac{1}{2}$ and $\frac{1}{3}$?
8. $\frac{1}{4}$ and $\frac{1}{6}$?
9. $\frac{2}{15}$ and $\frac{3}{10}$?
10. $\frac{13}{15}$ and $\frac{7}{20}$?
11. $\frac{7}{12}$, $\frac{9}{10}$ and $\frac{11}{36}$?
12. $\frac{25}{36}$, $\frac{16}{45}$ and $\frac{37}{70}$?

What is the fractional unit of lowest denomination to which can be reduced—

13. $\frac{2}{5}$ in., $\frac{12}{15}$ in., $\frac{20}{25}$ in. and $\frac{16}{20}$ in.?
14. $\frac{23}{60}$ hr., $\frac{17}{15}$ hr. and $\frac{9}{40}$ hr.?
15. $\frac{3}{4}$ oz., $\frac{6}{10}$ oz., $\frac{12}{15}$ oz. and $\frac{15}{12}$ oz.?

Reduce to equivalent fractions with least common denominator—

16. $\frac{5}{6}$ and $\frac{3}{8}$.
17. $\frac{7}{12}$ and $\frac{9}{15}$.
18. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.
19. $\frac{5}{6}$, $\frac{5}{9}$, $\frac{5}{12}$.
20. $\frac{1}{3}$, $\frac{3}{5}$, $\frac{5}{7}$, $\frac{7}{9}$.
21. $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$, $\frac{11}{12}$.
22. $\frac{7}{12}$, $\frac{11}{60}$, $\frac{5}{75}$, $\frac{18}{20}$.
23. $\frac{23}{48}$, $\frac{15}{27}$, $\frac{28}{32}$, $\frac{35}{56}$.
24. $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, $\frac{3}{10000}$.
25. $\frac{7}{10}$, $\frac{27}{60}$, $\frac{127}{250}$, $\frac{627}{1250}$.

Which is greater—

26. $\frac{7}{15}$ or $\frac{10}{21}$?
27. $\frac{19}{18}$ or $\frac{28}{27}$?
28. $\frac{17}{24}$ or $\frac{17+6}{24+6}$?
29. $\frac{17}{24}$ or $\frac{17-6}{24-6}$?

Find the greatest and the least of—

30. $\frac{37}{25}, \frac{37+5}{25+5}, \frac{37-5}{25-5}$.
31. $\frac{17}{25}, \frac{17+5}{25+5}, \frac{17-5}{25-5}$.
32. $1\frac{5}{4}$, $3\frac{3}{8}$, $3\frac{5}{7}$.
33. $5\frac{5}{12}$, $3\frac{8}{7}$, $5\frac{4}{6}$.

34. Of $\frac{17}{35}$, $\frac{23}{46}$ and $\frac{28}{56}$, which is intermediate in value?

35. Find a fraction intermediate in value to $\frac{2}{3}$ in. and $\frac{3}{4}$ in. with 24 as denominator.

36. Find a fraction intermediate in value to $\frac{11}{15}$ hr. and $\frac{7}{10}$ hr. with denominator 60.

37. Arrange in order of magnitude—

$\frac{0}{1}$, $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$.

III. ADDITION OF FRACTIONS.

Example—Find the sum of $\frac{3}{8}$, $\frac{7}{8}$ and $\frac{5}{8}$.

Take three slips of paper, equal to one another in length and in breadth, and cut them across, each into 8 equal pieces or eighths. Take 3 of the eighths of the first slip, 7 of those of the second slip, and 5 of those of the third slip, and put them all together. There will be 3+7+5=15 pieces or eighths, enough to make up one whole slip and leave 7 pieces over. Written in symbols, all this is—

$$\frac{3}{8}+\frac{7}{8}+\frac{5}{8}=\frac{3+7+5}{8}=\frac{15}{8}=1\frac{7}{8}.$$

If two or more fractions to be added together have a common denominator, *add the numerators together for the numerator of the sum and take the common denominator for its denominator.*

EXERCISE XLVIII.

Show by cutting slips of paper or pieces of twine that—

1. $\frac{3}{4}+\frac{3}{4}=\frac{3+3}{4}=\frac{6}{4}=1\frac{2}{4}=1\frac{1}{2}$.
2. $\frac{1}{8}+\frac{3}{8}+\frac{7}{8}=\frac{1+3+7}{8}=\frac{11}{8}=1\frac{3}{8}$.
3. $\frac{1}{6}+\frac{3}{6}+\frac{5}{6}+\frac{2}{6}=\frac{1+3+5+2}{6}=\frac{11}{6}=1\frac{5}{6}$.

Find the value of—

4. $\frac{1}{4}+\frac{3}{4}+\frac{2}{4}$.
5. $\frac{1}{3}+\frac{1}{3}+\frac{2}{3}$.
6. $\frac{3}{5}+\frac{2}{5}+\frac{4}{5}$.
7. $\frac{3}{11}+\frac{5}{11}+\frac{7}{11}+\frac{7}{11}$.
8. $1\frac{1}{7}+2\frac{2}{7}+3\frac{4}{7}$.
9. $3\frac{5}{8}+\frac{11}{8}+\frac{7}{8}$.
10. $\frac{7}{6}+\frac{11}{6}+\frac{13}{6}+\frac{17}{6}$.
11. $\frac{7}{16}+\frac{17}{16}+\frac{27}{16}+\frac{37}{16}$.
12. $\frac{37}{100}+\frac{29}{100}+\frac{37}{100}+6\frac{11}{100}$.

Find the sum of—

13. 6 pair, 3 pair and 2 pair, *or* 6(2), 3(2) and 2(2).
14. 6 doz., 3 doz. and 2 doz,, *or* 6(12), 3(12) and 2(12).
15. 6 score, 3 score and 2 score *or* 6(20), 3(20) and 2(20).
16. 6 hundred, 3 hundred and 2 hundred, *or* 6(100), 3(100) and 2(100).
17. 6 halves, 3 halves and 2 halves, *or* $\frac{6}{2}$, $\frac{3}{2}$ and $\frac{2}{2}$.
18. 6 quarters, 3 quarters and 2 quarters, *or* $\frac{6}{4}$, $\frac{3}{4}$ and $\frac{2}{4}$.
19. 6-eighths, 3-eighths and 2-eighths, *or* $\frac{6}{8}$, $\frac{3}{8}$ and $\frac{2}{8}$.
20. 6-fifths, 3-fifths and 2-fifths, *or* $\frac{6}{5}$, $\frac{3}{5}$ and $\frac{2}{5}$.
21. 6-thirty-fifths, 3-thirty-fifths and 2-thirty-fifths *or* $\frac{6}{35}$, $\frac{3}{35}$ and $\frac{2}{35}$.

Ex. 1.—Add together $\frac{3}{4}$ and $\frac{5}{6}$.

Take two slips of paper equal to each other in length and in width. Cut the first slip across into 4 equal pieces or quarters, and the second slip into 6 equal pieces or sixths. Take 3 of the pieces of the first slip and 5 of those of the second slip. You will now have 8 pieces, but they are neither all quarters nor all sixths; in fact, since they are not all of the same size, *they cannot be called 8 fractional parts of any one denomination.*

But quarters and sixths can both be reduced to twelfths (see Exercise XLVII.) the quarters by dividing each into 3 equal parts, and the sixths by dividing each into 2 equal parts. Reducing the quarters and the sixths in this way, the 3 quarters give $3\times3=9$-twelfths, and the 5 sixths give $5\times2=10$-twelfths. There are now $9+10=19$ pieces *all of the same length*, namely, one twelfth the length of a slip—that is, there are $\frac{19}{12}$ of a slip. Written in symbols, all this is—

$$\frac{3}{4}=\frac{3\times3}{4\times3}=\frac{9}{12}\qquad \frac{5}{6}=\frac{5\times2}{6\times2}=\frac{10}{12}$$

$$\frac{3}{4}+\frac{5}{6}=\frac{3\times3}{4\times3}+\frac{5\times2}{6\times2}=\frac{9+10}{12}=\frac{19}{12}=1\frac{7}{12}.$$

Ex. 2.—Find the value of $9\frac{2}{3}+2\frac{5}{8}+6\frac{11}{12}+1\frac{7}{15}$.

L. C. M. of denominators 3, 8, 12 and 15 is 120.

	120 ÷	3	8	12	15	Given Denominators.
		40	15	10	8	Quotients.
		2	5	11	7	Given Numerators.

$$\frac{2}{3}+\frac{5}{8}+\frac{11}{12}+\frac{7}{15}=\frac{80+75+110+56}{120}=\frac{321}{120}=2\frac{81}{120}=2\frac{27}{40}$$

$$9+2+6+1+2\tfrac{27}{40}=20\tfrac{27}{40}.$$

To find the sum of two or more fractions,

Reduce the fractions, if necessary, to a common denominator, add the resulting numerators together for the numerator of the sum, and take the common denominator for its denominator. Reduce the sum to its lowest terms, and if it be an improper fraction, reduce it to a mixed number.

If there be mixed numbers among the addends, add the fractional parts along with any fractional addends, and to the sum add the integral parts of the mixed number.

If there be improper fractions among the addends, reduce them to mixed numbers.

EXERCISE XLIX.

Show by dividing lines drawn on your slate, or by cutting slips of paper or pieces of twine, that—

1. $\frac{1}{2}+\frac{1}{4}=\frac{1\times 2}{2\times 2}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$.

2. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{1\times 3}{2\times 3}+\frac{1\times 2}{3\times 2}+\frac{1}{6}=\frac{3+2+1}{6}=\frac{6}{6}=1$.

3. $\frac{3}{4}+\frac{5}{6}+\frac{2}{3}=\frac{3\times 3}{4\times 3}+\frac{5\times 2}{6\times 2}+\frac{2\times 4}{3\times 4}=\frac{9+10+8}{12}=\frac{27}{12}=2\frac{3}{12}=2\frac{1}{4}$.

4. $3\frac{3}{4}+1\frac{1}{2}+\frac{4}{3}=3+1+1+\frac{3\times 3}{4\times 3}+\frac{1\times 6}{2\times 6}+\frac{1\times 4}{3\times 4}=5+\frac{19}{12}=6\frac{7}{12}$.

Add together—

5. $\frac{1}{2}$ and $\frac{3}{4}$. 6. $\frac{2}{3}$ and $\frac{5}{6}$. 7. $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{6}$. 8. $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{4}{6}$.

Find the sum of—

9. $1\frac{2}{3}$, $2\frac{3}{4}$ and $3\frac{4}{9}$.
10. $3\frac{1}{5}$, $5\frac{1}{8}$ and $2\frac{3}{10}$.
11. $1\frac{3}{12}$, $\frac{3}{15}$ and $3\frac{9}{20}$.
12. $3\frac{5}{21}$, 4 and $\frac{8}{35}$.

Find the value of—

13. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$.
14. $\frac{1}{2}+\frac{2}{8}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}$.
15. $\frac{2}{3}+\frac{5}{12}+\frac{13}{15}+\frac{1}{20}$.
16. $\frac{17}{12}+\frac{27}{20}+\frac{37}{24}+\frac{47}{30}$.
17. $\frac{5}{52}+\frac{5}{39}+\frac{5}{26}+\frac{5}{3}$.
18. $\frac{29}{20}+\frac{29}{10}+1\frac{19}{12}+\frac{19}{15}$.
19. $\frac{1}{8}+\frac{9}{24}+\frac{25}{40}+\frac{49}{80}$.
20. $\frac{12}{48}+\frac{35}{84}+\frac{77}{132}+\frac{143}{156}$.

21. $\$1\frac{1}{2}+\$3\frac{3}{4}+\$4\frac{4}{5}+\$7\frac{7}{10}+\$17\frac{17}{20}$.

22. $1\frac{4}{5}$ mi. $+3\frac{7}{11}$ mi. $+2\frac{3}{8}$ mi. $+9\frac{11}{18}$ mi.

23. $\frac{3}{11}$A. $+\frac{5}{9}$ A. $+\frac{8}{33}$ A. $+\frac{17}{363}$ A. $+\frac{15}{16}$ A.

24. $\frac{2}{7}$ bu. $+\frac{5}{14}$ bu. $+\frac{8}{35}$ bu. $+\frac{11}{56}$ bu. $+\frac{9}{70}$ bu.

25. A man bought at different times four lots containing respectively $\frac{2}{5}$, A., $\frac{3}{4}$ A., $\frac{1}{3}$ A. and $\frac{5}{8}$ A. How much land did he buy altogether?

26. The difference in weight between two boxes of tea is $17\frac{11}{16}$ lb., and the lighter box weighs $49\frac{9}{16}$ lb. What is the weight of the heavier?

27. The first of four measures holds $2\frac{1}{4}$ qt., the second holds $1\frac{2}{3}$ qt. more than the first, the third holds $\frac{7}{8}$ qt. more than the second, and the fourth holds $\frac{1}{6}$ qt. more than the first and the second together. How much do all four hold?

IV. SUBTRACTION OF FRACTIONS.

If the minuend and the subtrahend have a common denominator, and the numerator of the subtrahend be not greater than that of the minuend, the question is one of simple subtraction.

EXERCISE L.

From	take	*Written in symbols.*
1. 7 pair	3 pair;	$7(2) - 3(2)$.
2. 7 dozen	3 dozen;	$7(12) - 3(12)$.
3. 7 score	3 score;	$7(20) - 3(20)$.
4. 7 hundred	3 hundred;	$7(100) - 3(100)$.
5. 7-quarters	3-quarters;	$\frac{7}{4} - \frac{3}{4}$.
6. 7-eighths	3-eighths;	$\frac{7}{8} - \frac{3}{8}$.
7. 7-tenths	3-tenths;	$\frac{7}{10} - \frac{3}{10}$.
8. 7-twelfths	3-twelfths;	$\frac{7}{12} - \frac{3}{12}$.

Find the value of—

9. $\frac{8}{15} - \frac{4}{15}$. *11.* $\frac{37}{100} - \frac{19}{100}$. *13.* $7\frac{377}{1000} - 3\frac{139}{1000}$.

10. $\frac{17}{20} - \frac{8}{20}$. *12.* $\frac{49}{121} - \frac{21}{121}$. *14.* $37\frac{475}{1025} - 9\frac{89}{1025}$.

What proper fractions added to the following will in each case give an integral sum?

15. $\frac{1}{4}$. *18.* $\frac{21}{88}$. *21.* $7\frac{39}{75}$. *24.* $93\frac{739}{4000}$.

16. $\frac{3}{8}$. *19.* $\frac{137}{303}$. *22.* $91\frac{31}{200}$. *25.* $111\frac{114}{850}$.

17. $\frac{5}{7}$. *20.* $\frac{79}{100}$. *23.* $4\frac{400}{1001}$. *26.* $909\frac{803}{9007}$.

PRINCIPLE.—ADDING THE SAME NUMBER TO BOTH MINUEND AND SUBTRAHEND DOES NOT CHANGE THE DIFFERENCE OR REMAINDER.

To find the difference between two fractions,

Add to both minuend and subtrahend the proper fraction whose sum with the subtrahend is integral; then subtract the now integral subtrahend from the integral part of the minuend.

Ex. 1.—From $3\frac{1}{10}$ take $1\frac{7}{10}$.

$3\frac{1}{10} + \frac{3}{10} = 3\frac{4}{10}$

$1\frac{7}{10} + \frac{3}{10} = 2$ This line to be completed *first*.

$3\frac{1}{10} - 1\frac{7}{10} = 1\frac{4}{10} = 1\frac{2}{5}$

Ex. 2.—Find the value of $7\frac{3}{16} - 2\frac{3}{36}\frac{1}{6}$.

L. C. M. of 16 and 36 is 144.

$$7\frac{3}{16} + \frac{5}{36} = 7\frac{27+20}{144}$$

$$2\frac{31}{36} + \frac{5}{36} = 3$$

$$7\frac{3}{16} - 2\frac{31}{36} = 4\frac{47}{144}$$

EXERCISE LI.

Find the value of—

1. $3\frac{1}{6}\frac{3}{4} - \frac{3}{6}\frac{3}{4}$.
2. $20\frac{7}{10} - 3\frac{9}{10}$.
3. $17\frac{7}{45} - 11\frac{1}{45}$.
4. $15\frac{5}{28} - 14\frac{1}{28}$.
5. $6\frac{4}{4}\frac{9}{4}\frac{0}{1} - \frac{3}{4}\frac{4}{4}\frac{0}{1}$.
6. $13\frac{3}{35} - 3\frac{3}{35}$.
7. $\frac{1}{2} - \frac{1}{3}$.
8. $\frac{1}{3} - \frac{1}{4}$.
9. $\frac{3}{4} - \frac{2}{3}$.
10. $\frac{3}{5} - \frac{4}{7}$.
11. $\frac{4}{7} - \frac{3}{10}$.
12. $\frac{17}{18} - \frac{11}{20}$.
13. $\frac{1}{5} - \frac{10}{105}$.
14. $\frac{107}{144} - \frac{49}{120}$.
15. $\frac{1}{7} - \frac{17}{143}$.
16. $9\frac{2}{3} - 4\frac{4}{5}$.
17. $7\frac{3}{4} - 6\frac{3}{8}$.
18. $8\frac{1}{5} - 4\frac{2}{25}$.
19. $71\frac{7}{72} - 11\frac{1}{80}$.
20. $93\frac{1}{64} - 39\frac{3}{40}$.
21. $47\frac{47}{100} - 7\frac{7}{10}$.
22. $49\frac{9}{91} - 39\frac{9}{77}$.
23. $235\frac{3}{5} - 75\frac{3}{100}$.
24. $175\frac{3}{100} - 35\frac{3}{5}$.

25. By how much is $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$ greater than $\frac{1}{3}+\frac{1}{5}+\frac{1}{7}$?

26. By how much is $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$ less than $\frac{2}{3}+\frac{2}{5}+\frac{2}{7}$?

27. By how much does $\frac{3}{8} - \frac{3}{10}$ exceed $\frac{5}{12} - \frac{5}{14}$?

28. How much must be added to $\frac{1}{5} - \frac{1}{7}$ to give $\frac{1}{8}$ as sum?

29. How much must be taken from $\frac{1}{5}+\frac{1}{7}$ to leave $\frac{1}{8}+\frac{1}{8}$ as remainder?

30. Find the difference between $\frac{1}{5}+\frac{1}{8}$ and $\frac{1}{4}+\frac{1}{7}$.

31. Smith owns $\frac{3}{8}$ of a section of land; Jones owns $\frac{5}{12}$ of the section, and Brown owns the remainder of it. What fraction of the section does Brown own?

32. A teacher expended $\frac{1}{3}$ of his salary for board, $\frac{1}{5}$ of it for clothing, $\frac{1}{8}$ of it for books, and $\frac{3}{10}$ of it for other purposes. How much of his salary had he then left?

33. A piece of cloth measured $23\frac{1}{7}$ yd. before fulling, but only $21\frac{5}{6}$ yd. after fulling. How much did the cloth shrink?

34. Wilson agreed to sell $37\frac{1}{2}$ cords of wood to Jackson. He delivered $9\frac{1}{4}$ cords one week, $8\frac{7}{8}$ the next, and $10\frac{5}{8}$ the next. How many cords has he still to deliver?

V. MULTIPLICATION OF FRACTIONS.

Multiplication is the operation by which we find in terms of a proposed unit the value of a number whose unit is itself a number expressed in terms of the proposed unit.

Example.—How much is 7 times $\frac{3}{4}$?

7 times 3 quarters = (7 times 3) quarters = 21 quarters,

or writing the denomination *quarters* in symbols,

$$7 \text{ times } \frac{3}{4} = \frac{7 \text{ times } 3}{4} = \frac{21}{4}.$$

Using *multiplied by* instead of *times*, this becomes

3 quarters multiplied by 7 = (3 multiplied by 7) quarters = 21 quarters,

or, in symbols, $\frac{3}{4} \times 7 = \frac{3 \times 7}{4} = \frac{21}{4}$.

EXERCISE LII.

Find the value of—

1. 3 times $\frac{3}{4}$ in.
2. 5 times $\frac{3}{8}$ lb.
3. 3 times $\$\frac{7}{10}$.
4. 7 times $\$\frac{3}{10}$.
5. $\frac{3}{8}$ multiplied by 12.
6. $\frac{8}{21}$ multiplied by 14.
7. $\frac{5}{24}$ multiplied by 28.
8. $\frac{21}{16}$ multiplied by 24.
9. $\frac{15}{77} \times 22$.
10. $\frac{77}{15} \times 22$.
11. $\frac{17}{105} \times 60$.
12. $\frac{129}{115} \times 51$.

Ex. 1.—Find the value of $\frac{1}{3}$ of $\frac{3}{4}$ in.

Here we are required to find the value of $\frac{1}{3}$ of three things, these being each a quarter of an inch.

$\frac{1}{3}$ of 3 = 1;

hence, inserting the unit, namely, a quarter of an inch,

$\frac{1}{3}$ of $\frac{3}{4}$ in. = $\frac{1}{4}$ in.

Ex. 2.—Find $\frac{1}{5}$ of $\frac{3}{4}$ in.

$$\frac{1}{5} \text{ of } \frac{3}{4} \text{ in.} = \frac{1}{5} \text{ of } \frac{3 \times 5}{4 \times 5} \text{ in.} = \frac{3}{4 \times 5} \text{ in.} = \frac{3}{20} \text{ in.}$$

EXERCISE LIII.

Find the value of—

1. $\frac{1}{2}$ of $\frac{2}{3}$ yd.
2. $\frac{1}{4}$ of $\frac{1}{3}$ in.
3. $\frac{1}{3}$ of $\frac{2}{7}$ wk.
4. $\frac{1}{8}$ of $\frac{2}{3}$ ft.
5. $\frac{1}{6}$ of $\frac{3}{4}$ gal.
6. $\frac{1}{8}$ of $\frac{3}{8}$ bu.

Example.—Find the value of $\frac{7}{9}$ of $\frac{4}{11}$ rd.

ANALYSIS $\frac{1}{9}$ of $\frac{4}{11}$ rd. $= \frac{1}{9}$ of $\frac{4\times9}{11\times9}$ rd. $= \frac{4}{11\times9}$ rd.;

therefore $\frac{7}{9}$ of $\frac{4}{11}$ rd. $= \frac{4}{11\times9}$ rd. $\times 7 = \frac{4\times7}{11\times9}$ rd. $= \frac{28}{99}$ rd.

CALCULATION. $\frac{7}{9}$ of $\frac{4}{11}$ rd. $= \frac{4\times7}{11\times9}$ rd. $= \frac{28}{99}$ rd.

NOTE.—Abstract factors may be multiplied in any order. We might therefore have written

$$\frac{7}{9} \text{ of } \frac{4}{11} \text{ rd.} = \frac{7\times4}{9\times11} \text{ rd.} = \frac{28}{99} \text{ rd.}$$

Should any factor occur in both numerator and denominator, it may be divided out of both terms without affecting the value of the result, the effect being merely to reduce the result to lower terms. This is called **Cancelling the Factor.** Thus,

$$\frac{9}{16} \text{ of } \frac{10}{21} = \frac{\overset{3}{\cancel{9}}\times\overset{5}{\cancel{10}}}{\underset{8}{\cancel{16}}\times\underset{7}{\cancel{21}}} = \frac{15}{56}.$$

Here 2 has been cancelled out of the 10 and the 16, and 3 cancelled out of the 9 and the 21. Had these factors not been cancelled, the result would have been $\frac{90}{336}$, which can be reduced to $\frac{15}{56}$ by dividing both terms by 6, which is the product of 2 and 3, the factors cancelled.

EXERCISE LIV.

Find the value of—

1. $\frac{2}{3}$ of $\frac{4}{5}$ bu.
2. $\frac{3}{4}$ of $\frac{4}{7}$ ft.
3. $\frac{4}{5}$ of $\frac{15}{16}$ lb.
4. $\frac{5}{12}$ of $\frac{3}{4}$ yd.
5. $\frac{1}{6}$ of $\frac{3}{5}$ hr.
6. $\frac{6}{35}$ of $\frac{14}{15}$ T.
7. $\frac{13}{14}$ of $\frac{21}{52}$.
8. $\frac{10}{21}$ of $\frac{35}{36}$.
9. $\frac{15}{44}$ of $\frac{44}{15}$.
10. $\frac{33}{35}$ of $\frac{42}{66}$.
11. $\frac{44}{91}$ of $\frac{39}{55}$.
12. $\frac{2}{3}$ of $1\frac{1}{2}$.
13. $\frac{15}{16}$ of $2\frac{2}{3}$.
14. $1\frac{1}{2}$ of $\frac{4}{5}$.
15. $1\frac{1}{2}$ of $2\frac{2}{3}$.
16. $3\frac{1}{3}$ of $\frac{4}{5}$.
17. $1\frac{1}{45}$ of $9\frac{1}{10}$.
18. $2\frac{21}{40}$ of $2\frac{203}{288}$.
19. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{3}{4}$.
20. $\frac{8}{11}$ of $\frac{20}{21}$ of $\frac{33}{44}$.
21. $\frac{3}{4}$ of $\frac{8}{45}$ of $\frac{24}{25}$.
22. $\frac{33}{35}$ of $\frac{42}{77}$ of $\frac{60}{88}$ of $2\frac{6}{33}$.
23. $\frac{33}{40}$ of $1\frac{119}{121}$ of $\frac{25}{42}$.
24. $1\frac{1}{2}$ of $2\frac{2}{3}$ of $3\frac{3}{4}$ of $4\frac{4}{5}$.

A fraction of a number—whether integral, mixed or fractional—is called a **Compound Fraction.**

Examples.—$\frac{3}{4}$ of 5, $\frac{2}{3}$ of $1\frac{1}{4}$, $\frac{3}{7}$ of $\frac{5}{6}$.

A **Compound Fraction** is therefore a *Fraction whose Prime Unit is itself a number.*

Now, the operation by which we find the value of a number whose unit is itself a number is called multiplication. Hence saying "$\frac{4}{5}$ multiplied by $\frac{2}{3}$" is merely another way of saying "$\frac{2}{3}$ of $\frac{4}{5}$." The following are other examples of different ways of expressing one and the same statement. [In these examples 1(2) is to be read *one-pair* and 4(12) read *four dozen*, just as $\frac{1}{2}$ is read one-half and $\frac{4}{12}$ is read four-twelfths.]

1. Seven of four-dozen each
   $=7$ times $4(12)=4(12)$ multiplied by $7=4(12)\times 7=28(12)$.

2. Three of five-sixths each
   $=3$ times $\frac{5}{6}=\frac{5}{6}$ multiplied by $3=\frac{5}{6}\times 3=\frac{15}{6}$.

3. Three-dozen pairs
   $=3(12)(2)=1(2)$ multiplied by $3(12)=1(2)\times 3(12)=3(24)$.

4. Two-dozen of five-pairs each
   $=2(12)$ of $5(2)=5(2)$ multiplied by $2(12)$
   $=5(2)\times 2(12)=10(24)$.

5. Two-thirds of four-fifths
   $=\frac{2}{3}$ of $\frac{4}{5}=\frac{4}{5}$ multiplied by $\frac{2}{3}=\frac{4}{5}\times\frac{2}{3}=\frac{8}{15}$.

EXERCISE LV.

Express the following products as compound fractions and find the value of each :—

1. $\frac{2}{3}$ ft. $\times \frac{1}{2}$.	*3.* $\frac{4}{5}$ pt. $\times \frac{7}{8}$.	*5.* $\frac{7}{12}$ ft. $\times 1\frac{5}{7}$
2. 3 in. $\times \frac{3}{4}$.	*4.* $3\frac{3}{4}$ lb. $\times \frac{2}{5}$.	*6.* $1\frac{7}{8}$ gal. $\times 1\frac{1}{4}$.

Express the following compound fractions as products and find the value of each :—

7. $\frac{4}{5}$ of $\frac{1}{2}$ hr.	*9.* $1\frac{1}{3}$ of $\frac{5}{8}$ oz.	*11.* $\frac{1}{10}$ of $3\frac{1}{3}$.
8. $\frac{3}{4}$ of $2\frac{1}{2}$ lb.	*10.* $3\frac{3}{4}$ of $2\frac{2}{3}$.	*12.* $3\frac{1}{3}$ of $3\frac{3}{10}$.

To simplify a compound fraction, or to find the product of factors one or more of which are fractions or mixed numbers,

If any of the fractions are mixed numbers, reduce these to equivalent improper fractions, and write integral factors in the form of fractions with 1 as denominator.

The product of the numerators of the factors will be the numerator of their product.

The product of their denominators will be the denominator of their product.

Factors common to both a numerator and a denominator should be cancelled.

EXERCISE LVI.

Find the value of—

1. $\frac{14}{34} \times \frac{9}{16} \times \frac{24}{30}$.
2. $\frac{4}{5} \times 1\frac{3}{5} \times 1\frac{9}{16}$.
3. $\frac{16}{35} \times \frac{9}{34} \times \frac{7}{8} \times \frac{17}{27}$.
4. $\frac{3}{4}$ of $8\frac{7}{9} \times \frac{2}{7}$ of $1\frac{5}{11}$.
5. $1\frac{1}{36} \times 1\frac{7}{30}$ of $3\frac{13}{25} \times 11\frac{14}{21}$.
6. $3\frac{1}{4}$ of $3\frac{1}{4}$ of $\frac{64}{65} \times \frac{1}{244}$.
7. $4\frac{4}{9}$ of $2\frac{2}{5}$ of $5\frac{1}{4} \times \frac{19}{38}$.
8. $\frac{12}{13} \times \frac{26}{25} \times \frac{28}{75} \times \frac{49}{64} \times \frac{76}{133} \times \frac{38}{98}$.
9. $3\frac{1}{8}$ of $7 \times 4\frac{1}{5}$ of $\frac{10}{343}$.
10. $\frac{1}{7}$ of $17 \times \frac{3}{11}$ of $63 \times \frac{44}{105}$.
11. Find the sum of $\frac{3}{4}$ of $\frac{5}{6}$ and $\frac{4}{5}$ of $\frac{6}{7}$.
12. Find the product of $\frac{3}{4} + \frac{2}{3}$ and $\frac{3}{4} - \frac{2}{3}$.
13. Find the nearest integer to the product of $3\frac{3}{4}$ and $12\frac{1}{2}$.
14. How far could a man walk in $2\frac{1}{2}$ hr. at the rate of $3\frac{1}{2}$ miles an hour?

Find the price, to the nearest cent, of—

15. $3\frac{1}{4}$ dozen eggs @ 17 ct. a doz.
16. $4\frac{1}{2}$ lb. tea @ 65 ct. a lb.
17. $3\frac{3}{4}$ lb. sugar @ $8\frac{1}{4}$ ct. a lb.
18. $17\frac{3}{4}$ yd. of calico at $11\frac{1}{2}$ ct. a yard.
19. $4\frac{1}{2}$ doz. tins of tomatoes @ $1.00 a doz.
20. $37\frac{1}{2}$ bu. oats @ $37\frac{1}{2}$ ct. a bu., and $45\frac{3}{8}$ bu. wheat @ $85\frac{7}{8}$ ct. a bu.
21. Find the weight of the water in a cistern containing $75\frac{3}{16}$ gal.
22. A man sold $17\frac{3}{4}$ gross of boxes of matches, gaining $3\frac{3}{8}$ ct. per doz. boxes. How much did he gain on the whole?
23. Bronze consists of 1 part of tin to $4\frac{1}{4}$ parts of copper. What weight of copper must be added to $1653\frac{1}{2}$ lb. of tin to make bronze?

VI. DIVISION OF FRACTIONS.

The **Reciprocal** of any given number is the number whose product with the given number is *one*. Thus $2 \times \frac{1}{2} = 1$; therefore $\frac{1}{2}$ is the reciprocal of 2, and 2 is the reciprocal of $\frac{1}{2}$. $\frac{3}{4} \times \frac{4}{3} = 1$; therefore $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$, and $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$. $3\frac{1}{3} \times \frac{3}{10} = 1$; therefore $\frac{3}{10}$ is the reciprocal of $3\frac{1}{3}$ or $\frac{10}{3}$, and $\frac{10}{3}$ or $3\frac{1}{3}$ is the reciprocal of $\frac{3}{10}$.

EXERCISE LVII.

Find the reciprocal of—

1. 3.	*4.* $\frac{1}{4}$.	*7.* $\frac{3}{8}$.	*10.* $17\frac{1}{17}$.
2. 5.	*5.* $\frac{1}{9}$.	*8.* $\frac{9}{10}$.	*11.* $1 - \frac{1}{3}$.
3. 12.	*6.* $\frac{2}{5}$.	*9.* $5\frac{1}{3}$.	*12.* $\frac{1}{2} + \frac{1}{3}$.

Division is the operation by which we find the number which, taken as cofactor with one of two given numbers, would yield the other given number, as product. (See page 28.)

Ex. 1.—Divide 4 by $\frac{1}{2}$.

SOLUTION.

$4 = 4 \times 2 \times \frac{1}{2}$

therefore $4 \div \frac{1}{2} = 4 \times 2 \times \frac{1}{2} \div \frac{1}{2}$

$= 4 \times 2 = 8.$

This is merely another way of asking how many halves are equal to 4, *or* what number multiplied by $\frac{1}{2}$ would be equal to 4. PROOF.—$8 \times \frac{1}{2} = 4$.

Ex. 2.—Divide 5 by $\frac{3}{7}$.

SOLUTION.

$5 = 5 \times \frac{7}{3} \times \frac{3}{7}$;

therefore $5 \div \frac{3}{7} = 5 \times \frac{7}{3} \times \frac{3}{7} \div \frac{3}{7}$

$= 5 \times \frac{7}{3} = \frac{35}{3}$

$= 11\frac{2}{3}$.

This is merely another way of asking how many $\frac{3}{7}$ are equal to 5, *or* what number multiplied by $\frac{3}{7}$ would be equal to 5. PROOF.—$11\frac{2}{3} \times \frac{3}{7} = \frac{35}{3} \times \frac{3}{7} = 5$.

Ex. 3.—$\frac{4}{5} \div \frac{9}{10}$.

$\frac{4}{5} \div \frac{9}{10} = \frac{4}{5} \times \frac{10}{9} \times \frac{9}{10} \div \frac{9}{10}$

$= \frac{4}{5} \times \frac{\overset{2}{\cancel{10}}}{9} = \frac{8}{9}.$ PROOF.—$\frac{8}{9} \times \frac{9}{10} = \frac{4}{5}$.

Ex. 4.—$1\frac{1}{39} \div 6\frac{2}{13}$.

$$1\tfrac{1}{39} \div 6\tfrac{2}{13} = \tfrac{40}{39} \div \tfrac{80}{13} = \tfrac{40}{39} \times \tfrac{13}{80} \times \tfrac{80}{13} \div \tfrac{80}{13}$$

$$= \frac{\cancel{40}}{\cancel{39}_{3}} \times \frac{\cancel{13}}{\cancel{80}_{2}} = \frac{1}{6}.$$

PROOF.—$\frac{1}{6} \times 6\frac{2}{13} = \frac{1}{6} \times \frac{80}{13} = \frac{40}{39} = 1\frac{1}{39}$.

From the preceding examples we may see that **to divide by a fraction** we may

Multiply the dividend by the reciprocal of the divisor.

EXERCISE LVIII.

Find the value of—

1. $6 \div \frac{1}{2}$.	*7.* $12 \div \frac{6}{5}$.	*13.* $3\frac{1}{3} \div 1\frac{2}{3}$.	*19.* $9\frac{9}{10} \div 1\frac{1}{10}$.
2. $6 \div \frac{2}{3}$.	*8.* $12 \div \frac{9}{5}$.	*14.* $4\frac{4}{5} \div \frac{9}{10}$.	*20.* $1\frac{1}{4} \div 9\frac{9}{10}$.
3. $12 \div \frac{1}{3}$.	*9.* $8 \div \frac{2}{3}$.	*15.* $\frac{3}{4} \div \frac{1}{3}$.	*21.* $3\frac{3}{4} \div 4\frac{1}{2}$.
4. $12 \div \frac{2}{3}$.	*10.* $5 \div \frac{3}{5}$.	*16.* $\frac{12}{25} \div \frac{3}{5}$.	*22.* $4\frac{1}{2} \div 3\frac{3}{4}$.
5. $12 \div \frac{3}{5}$.	*11.* $3\frac{1}{3} \div \frac{1}{3}$.	*17.* $\frac{51}{77} \div 1\frac{1}{33}$.	*23.* $\frac{99}{100} \div \frac{99}{1000}$.
6. $12 \div \frac{4}{5}$.	*12.* $3\frac{1}{3} \div \frac{2}{3}$.	*18.* $\frac{91}{111} \div 1\frac{15}{37}$.	*24.* $\frac{99}{100} \div \frac{100}{99}$.

25. A man distributed $535\frac{1}{2}$ lb. of flour among a number of poor persons, giving $14\frac{7}{8}$ lb. to each. How many received relief? Had there been 2 persons fewer, how much more would each assisted person have received?

26. From a heap of shot weighing $7\frac{3}{8}$ lb., 3465 shot are taken, and the heap is then found to weigh $4\frac{3}{4}$ lb. Find the weight of a single shot and the number originally in the heap.

27. Find the railway fare for 315 mi. at the rate of $1.60 for 56 miles.

28. Which is cheaper, eggs bought at the rate of 7 for 10 ct. or at 17 ct. per doz.? How much would be gained on 150 doz. eggs bought at the cheaper rate and sold at the dearer?

29. How many pounds of butter @ $18\frac{3}{4}$ ct. a lb. will pay for $43\frac{1}{2}$ lb. of sugar @ $8\frac{1}{4}$ ct. a lb.? (Reckon to nearest ounce.)

30. Gun-metal is composed of 1 part of tin to $5\frac{1}{2}$ parts of copper. What weight of tin must be added to $420\frac{3}{4}$ lb. of copper to make gun-metal?

31. How many pounds of copper would there be in $464\frac{3}{4}$ lb. of gun-metal composed of 1 part of tin to $5\frac{1}{2}$ parts of copper?

The division of 6 by 3 may be expressed either by $6 \div 3$ or by $\frac{6}{3}$; the division of 8 by 12 may be expressed either by $8 \div 12$ or by $\frac{8}{12}$. In like manner the division of $2\frac{1}{2}$ by $3\frac{3}{4}$ may be expressed either by $2\frac{1}{2} \div 3\frac{3}{4}$ or by $\frac{2\frac{1}{2}}{3\frac{3}{4}}$, read $2\frac{1}{2}$ *divided by* $3\frac{3}{4}$. Such a fraction as $\frac{2\frac{1}{2}}{3\frac{3}{4}}$ is called a *complex fraction.*

A **Complex Fraction** *is a fraction one or both of whose terms contain a fraction.*

Since a complex fraction merely denotes the division of its numerator by its denominator,

The value of a complex fraction is the product of its numerator by the reciprocal of its denominator.

Ex. 1.—Reduce $\frac{\frac{1}{2}}{\frac{2}{3}}$ to a simple fraction—*that is,* find its value.

$$\frac{\frac{1}{2}}{\frac{2}{3}} = \tfrac{1}{2} \div \tfrac{2}{3} = \tfrac{1}{2} \times \tfrac{3}{2} = \tfrac{3}{4}.$$

Ex. 2.—Reduce $\frac{2\frac{1}{2}}{3\frac{3}{4}}$ to a simple fraction.

$$\frac{2\frac{1}{2}}{3\frac{3}{4}} = \frac{\frac{5}{2}}{\frac{15}{4}} = \tfrac{5}{2} \times \tfrac{4}{15} = \tfrac{2}{3}.$$

EXERCISE LIX.

Read—

1. $\frac{\frac{2}{3}}{\frac{5}{4}}$.
2. $\frac{3\frac{3}{4}}{4}$.
3. $\frac{4}{3\frac{3}{4}}$.
4. $\frac{4\frac{1}{2}}{8\frac{1}{10}}$.

Express as complex fractions—

5. $3\frac{1}{2} \div 4\frac{2}{3}$.
6. $\frac{3}{4} \div \frac{9}{16}$.
7. $24 \div 5\frac{1}{3}$.
8. $\frac{5}{9} \div 10$.
9. $\frac{1}{2} - \frac{1}{3}$ divided by $\frac{1}{2} + \frac{1}{3}$.
10. $\frac{1}{2}$ of $\frac{3}{4}$ divided by $\frac{1}{2} \div \frac{3}{4}$.

11 to *20.* Find the value of each of the preceding ten complex fractions.

Simplify—

21. $\frac{\frac{5}{8}}{6}$.
22. $\frac{6}{\frac{5}{8}}$.
23. $\frac{\frac{8}{9}}{12}$.
24. $\frac{8}{\frac{9}{12}}$.
25. $\frac{\frac{2}{3} - \frac{1}{6}}{3}$.
26. $\frac{\frac{2}{3} + \frac{5}{6}}{1\frac{1}{3}}$.
27. $\frac{1\frac{1}{5} - \frac{3}{10}}{1\frac{4}{5} + \frac{3}{10}}$.
28. $\frac{2\frac{1}{2} \times \frac{5}{8}}{2\frac{1}{2} - \frac{5}{8}}$.
29. $\frac{1 - \frac{2}{3} \text{ of } \frac{4}{5}}{1 - \frac{1}{3} \text{ of } \frac{1}{5}}$.
30. $\frac{9 \div 1\frac{5}{7}}{8\frac{3}{11} \times 2\frac{7}{13}}$.

VII. DENOMINATE FRACTIONS.

EXERCISE LX.

1. How many ounces are there in $3\frac{3}{4}$ lb.?
2. Reduce $\frac{7}{8}$ T. to pounds.
3. Express $\frac{8}{9}$ mi. in yd., ft., in.
4. Express $\frac{7}{15}$ A. in sq. yd. and sq. ft.

Reduce—

5. $\frac{3}{16}$ cu. yd. to cu. ft., etc.
6. $\frac{3}{8}$ gal. to qt., etc.
7. $\frac{7}{24}$ bu. to pk., etc.
8. $\frac{23}{400}$ da. to hr., etc.

Find—

9. $\frac{4}{7}$ of 5 lb.
10. $\frac{4}{7}$ of 3 T.
11. $\frac{5}{9}$ of $5\frac{1}{2}$ mi.
12. $\frac{4}{11}$ of 1 mi. 1 rd.
13. $\frac{13}{30}$ of 2 A. 620 sq. yd.
14. $\frac{13}{73}$ of 3 yr. 3 da. 2 hr.
15. 4 cords 24 cu. ft. $\times 4\frac{9}{16}$.
16. 54 gal. 3 qt. $\times 15\frac{16}{16}$.
17. To $3\frac{3}{5}$ lb. add $12\frac{3}{8}$ oz.
18. Add together $\frac{4}{5}$ yd., $\frac{3}{9}$ ft. and $\frac{1}{16}$ in.
19. Find the sum of $\frac{1}{8}$ bu., $\frac{3}{4}$ pk., $\frac{5}{8}$ gal. and $\frac{3}{8}$ of 5 bu. 1 gal. 2 qt. 1 pt.
20. To the sum of $\frac{13}{35}$ of 3 A. 2420 sq. yd. and $\frac{3}{13}$ of 1 A. $428\frac{1}{4}$ sq. yd. add the difference between 13 A. $3\frac{1}{3}$ sq. rd. and $7\frac{2}{3}\frac{3}{4}$ A.
21. From $3\frac{3}{4}$ lb. take $33\frac{3}{4}$ oz.
22. What length added to $\frac{11}{7}$ yd. will make $\frac{7}{11}$ rd.?
23. Find the difference between $\frac{3}{5}$ sq. mi. and $\frac{5}{8}\frac{3}{3}$ of 1000 A.
24. By how much is $\frac{5}{17}$ of 5 da. longer than $\frac{5}{17}$ of $33\frac{1}{3}$ hr.?
25. Subtract $43\frac{1}{2}$ times 45 cu. ft. from $\frac{4}{5}$ of $43\frac{1}{2}$ cords.

Divide—

26. 3 T. 400 lb. by $7\frac{1}{6}$.
27. 3 mi. 720 yd. by $\frac{3}{8}\frac{3}{7}$.

Find the quotient of—

28. 4 A. 2360 sq. yd. $\div \frac{5}{10}$.
29. 17 bu. 3 pk. 1 gal. $\div 3\frac{3}{5}$.
30. 24 cu. ft. is $4\frac{1}{2}\frac{1}{6}$ times a certain volume; find that volume.
31. Find the length of time of which 36 da. 2 hr. is $\frac{5}{16}$.
32. Divide \$45 by $3+\frac{3}{4}$.
33. Divide 7 lb. 1200 gr. by $1\frac{1}{2}+2\frac{2}{5}-3\frac{3}{4}$.

34. What fraction of a pound is 4 oz.?

35. Express 37 oz. in pounds.

36. How much of 1 yd. is 6 in.?

37. Reduce 2 ft. 5 in. to the fraction of a yard.

38. What fraction of a mile is 3 rd. 1 yd.?

39. Express 1127 rd. 2 ft. 3 in. in miles.

40. What fraction of an acre is 1239 sq. rd. 13 sq. yd.?

41. How much of a dollar is $2\frac{1}{2}$ ct.?

42. What fraction of a dollar is $1\frac{7}{8}$ ct.?

Express in bushels—

43. 1115 lb. of wheat.

44. 1616 lb. of barley.

45. 1966 lb. of oats.

46. 1477 lb. of Indian corn.

47. 1640 lb. of buckwheat.

48. 1840 lb. of peas.

49. 1480 lb. of timothy seed.

50. 1370 lb. of red clover seed.

Divide—

51. $3\frac{3}{4}$ lb. by $4\frac{1}{2}$ oz.

52. 4 mi. 480 yd. by $1\frac{1}{4}$ mi.

53. 300 sq. yd. by $1\frac{1}{8}$ A.

54. $\frac{3}{8}$ yd. by $\frac{5}{9\,9}$ mi.

Find the quotient of—

55. $\frac{3}{4}$ lb. $\div \frac{5}{8}$ oz.

56. $\frac{4}{5}$ oz. $\div \frac{3}{4}$ lb.

57. $22° 27\frac{3}{8}' \div 90°$.

58. Divide $\frac{5}{8}$ of $4\frac{6}{11}$ A. by $\frac{3}{4}$ of 25 sq. rd.

59. Divide $\frac{3}{8}$ of 3 gal. $1\frac{3}{4}$ qt. by $\frac{6}{13}$ of 2 bu. 3 pk.

60. What fraction of 1 cwt. is $37\frac{1}{2}$ lb.?

61. What fraction of $9\frac{1}{4}$ A. is 1628 sq. yd.?

62. What fraction of $6\frac{1}{4}$ mi. is 375 yd.?

63. Reduce 17 da. 3 hr. to the fraction of $365\frac{1}{4}$ da.?

64. Express 2763 lb. of wheat as a fraction of 63 bu.?

65. Express $\frac{7}{11}$ of 13 mi. 3 rd. as a fraction of 20 mi.

66. $\frac{4}{11\,5}$ of $31\frac{1}{2}$ gal. of water is what fraction of $\frac{7}{8}$ cu. yd. of water?

67. What fraction of 5 T. is $7\frac{1}{2}$ bu. of soft coal?

68. The profits of a certain business are divided into 104 equal parts, and I receive $19\frac{1}{2}$ of these parts. What fraction of the profits do I receive?

69. How many twelfths of an inch are there in $2\frac{2}{3}$ ft.?

70. How often is the third of an inch contained in $\frac{2}{11}$ of $\frac{5}{4\,0}$ mi.?

71. How many lengths of $3\frac{1}{5}$ yd. each are there in $44\frac{3}{4}$ yd., and what fraction of a length would there be over?

72. How many kegs, each holding $3\frac{4}{11\,5}$ gal., could be filled from two barrels, one containing $27\frac{3}{8}$ gal. and the other $30\frac{3}{16}$ gal.?

73. How many bottles, each holding $\frac{4}{5}$ qt., would $9\frac{3}{4}$ bbl. of vinegar fill, reckoning $31\frac{1}{2}$ gal. to the full bl., and what fraction of a bottleful would there be over?

Express as the fraction of a year (365 da.)—

74. From noon of 3rd April, 1886, to noon of 24th Aug., 1886.

75. From noon of 17th May, 1886, to noon of 5th Dec., 1886.

76. From noon of 19th Dec., 1884, to noon of 14th Dec., 1885.

77. From noon of 23rd Oct., 1887, to noon of 12th May, 1888.

78. A farmer sold 234 bu. of his wheat crop and kept for his own use the 78 bushels remaining. What fraction of his wheat crop did he sell?

79. A man who had \$42 spent \$2.10 of that sum. What fraction of his money did he spend? What fraction of it had he remaining?

80. A man bought a horse for \$80 and sold him for \$96. What fraction of the cost of the horse did he gain?

81. Smith bought a horse for \$120 and sold him to Jones for \$150. Jones next sold the horse to Brown for \$120. What fraction of the cost of the horse to him did Smith gain? What fraction of the cost of the horse to him did Jones lose?

82. A certain mine yields 113 lb. 5 oz. of metal from every $7\frac{3}{16}$ T. of ore. What fraction of the ore is the metal extracted? What weight of metal ought 274 T. 1120 lb. to yield?

83. If $7\frac{1}{2}$ A. yield $101\frac{1}{4}$ bu. of wheat, how many bushels would 15 A. 1760 sq. yd. yield at the same rate?

84. Armstrong has \$7.56 and Brown has \$12. Armstrong gives $\frac{1}{4}$ of his money to Brown, and then \$2.10 more. What fraction of his (Armstrong's) money did Armstrong give in all to Brown? After Brown had received the money, what fraction of what he then had had he received from Armstrong?

85. Allan has \$10.20; Barnes has \$24.50. Allan lends Barnes \$1.10 more than a third of his (Allan's) money. Next day Barnes, who has meanwhile spent \$1.50, repays Allan. What fraction of his (Barnes') money has he to give Allan to repay him?

86. A cistern can be filled by a pipe in 15 hr. How much of the cistern could be filled in 3 hr.? In $3\frac{3}{4}$ hr.? In 4 hr. 20 min.?

87. When the tap is open, $\frac{3}{4}$ of a cistern is filled in $4\frac{1}{2}$ hr.? At that rate how long would it take to fill the cistern? How long would it take to fill $\frac{1}{5}$ of the cistern? What fraction of the cistern would be filled in $3\frac{1}{4}$ hr.?

88. From the end of a plank 14 ft. $7\frac{1}{2}$ in. long $2\frac{1}{8}$ of $\frac{4}{9}$ of the whole is cut away. What length remains?

89. Three persons received respectively a fifth, a sixth and an eighth of $14.40. What sum remained? What fraction was it of the whole?

90. I owe $15.75 to Fraser, 2.99\frac{1}{2}$ more than half as much to May, and to Graham 2.99\frac{1}{2}$ less than half as much again as I owe to May. How much less than $50 do I owe to Fraser, May and Graham together? What fraction of $99 would it require to pay the whole of these debts?

91. A school-room is half as long again as it is wide. What fraction of the perimeter is the width?

92. A man had to walk 10 miles. He walked $\frac{1}{3}$ of the way, rested one hour, and then walked 2 mi. 720 yd. What fraction of his journey had he still to walk?

93. A man made a journey of 100 miles. He rode 7 mi. 340 yd., travelled by rail $\frac{4}{5}$ of the remainder of the way, and made all but 1250 yd. of what still remained of his trip by steamboat. What fraction of his trip was made by steamboat?

94. In constructing a sewer 104,650 bricks were supplied, and out of this number 96,600 were used and the rest rejected. What fraction of the whole did the rejected bricks form?

95. B is older than A by $\frac{1}{4}$ of *A*'s age which is 54 yr. Find B's age and express the difference between the ages of A and B as a fraction of B's age.

96. What is the difference between eleven times three-quarters of $\frac{1}{4}$ of $3\frac{3}{8}$ mi. and three times four elevenths of 7680 rd.?

97. How many steps, each having a $5\frac{3}{4}$ in. riser, would be required for a staircase reaching a perpendicular height of 12 ft.? What height would have to be *distributed* to make the exact 12 ft.?

98. A geographical mile is the $\frac{1}{60}$ of $\frac{1}{360}$ part of the earth's circumference. The equatorial circumference is 131,483,200 ft. How many common or statute miles are equal to 60 geographical miles on the equator?

99. A nautical mile contains 1000 fathoms of 6 ft. each. How many statute miles are equal to 60 nautical miles?

100. The area of Greece is $\frac{3}{11}$ of that of Britain. Spain has $2\frac{1}{6}$ times the area Britain has. What fraction of the area of Spain is the area of Greece?

101. A man left $\frac{1}{4}$ of his property to his eldest son, $\frac{1}{8}$ of it to each of his two other sons, $\frac{1}{5}$ of it to each of his three daughters, and the remaining $2500 to his brother. What was the value of the whole?

VIII. APPLICATIONS OF THE PRECEDING RULES.

EXERCISE LXI.

In commercial transactions reckon to the nearest cent; half a cent to be considered a whole cent.

Find the value of—

1. 4 lb. 9 oz. of butter @ 19 ct. a lb.
2. 8 lb. 7 oz. of mutton @ 11 ct. a lb.
3. $5\frac{1}{2}$ qt. of molasses @ \$1.15 a gal.
4. Two hams, one weighing 14 lb. 6 oz. and the other weighing 17 lb. 12 oz., both @ $16\frac{1}{4}$ ct. a lb.
5. 1430 lb. of wheat @ 93 ct. a bu.
6. 1887 lb. of oats @ $43\frac{1}{2}$ ct. "
7. 1795 lb. of wheat @ $89\frac{3}{8}$ ct. "
8. 1896 lb. of barley @ $63\frac{5}{8}$ ct. "
9. 1678 lb. of hay @ \$23.40 a T.

Make out bills for the following-stated transactions, supplying dates and names of places where necessary :—

10. Thos. Jones bought of E. B. Browne $3\frac{3}{4}$ lb. of Butter @ 21 ct. a lb., $2\frac{1}{2}$ doz. Eggs @ 15 ct. a doz., $\frac{1}{2}$ lb. Japan Tea @ 45 ct. a lb., $5\frac{3}{4}$ lb. Sugar @ 9 ct., $\frac{1}{4}$ lb. Peel @ 33 ct., $4\frac{3}{4}$ lb. Cheese @ $15\frac{1}{2}$ ct.

11. Messrs. Mason @ Wright sold to James Chamberlain, on May 1st, 1886, 240 lb. of Flour @ \$3.10 a cwt.; May 6th, $137\frac{1}{2}$ lb. of Oatmeal @ \$2.35; May 11th, 366 lb. of Cornmeal @ \$2.30; May 18th, 245 lb. of Buckwheat Flour @ \$2.45; May 28th, 330 lb. of Flour @ \$3.05. On 1st June, 1886, Mr. Chamberlain paid \$20 on this account to Timothy Webster, bookkeeper for Messrs. Mason & Wright. (Make out receipt for the payment.)

12. William Simpson bought of Alfred Spencer $6\frac{1}{2}$ lb. Veal @ $12\frac{1}{2}$ ct. a lb., $1\frac{1}{4}$ lb. Lard @ 17 ct., $1\frac{1}{2}$ lb. Bacon @ 15 ct., $7\frac{3}{4}$ lb. Corned Beef @ 9 ct., $6\frac{1}{4}$ lb. Lamb @ $11\frac{1}{2}$ ct., $3\frac{3}{4}$ lb. Steak @ $12\frac{1}{2}$ ct., $13\frac{3}{4}$ lb. Roasting Beef @ 14 ct.

13. Henry Mitchell sold to John Young, on April 12th, $15\frac{1}{4}$ yd. Print @ $11\frac{1}{4}$ ct., $11\frac{1}{2}$ yd. Silk @ \$2.25, $7\frac{1}{4}$ yd. Lining @ $12\frac{1}{2}$ ct.; April 17th, $9\frac{1}{4}$ yd. Tweed @ 97 ct.; $4\frac{1}{8}$ yd. Cloaking @ \$2.87$\frac{1}{2}$, $1\frac{7}{8}$ yd. Plush @ \$2.12$\frac{1}{2}$. $2\frac{1}{4}$ doz. Buttons @ 18 ct.; 9 Spools @ 50 ct. a doz. Paid in full to Henry Mitchell on April 30th.

14. On 1st September, 1886, Messrs. Bowes Bros. rendered an account to Egbert Henderson for the sum of $39.84. During the month the following items were added to the account: Sept. 4th, 17¼ yd. Calico @ 15 ct., 11⅜ yd. Lining @ 7½ ct.; 8th, 15¼ yd. Flannel @ 62½ ct.; 13th, 19 yd. Linen @ 37½ ct.; 21st, 4 pr. Gloves @ 85 ct., 9 pr. Stockings at 37½ ct., 7 pr. Socks @ 56 ct.; 28th, 17⅝ yd. Tweed @ $1.37½, 7¾ yd. Lining @ 23 ct.; 30th, ⅔ doz. Handkerchiefs @ $3.50. On September 30th the sum of $45 was paid on the account.

15. Adams bought 17 T. 640 lb. of soft coal @ $5 a ton and sold it @ 25 ct. per bu. Wilkinson bought an equal weight @ 17¼ ct. per bu. and sold it at $7.40 a ton. Which gained most and by how much?

16. Find the value of 1672 sq. yd. @ $135 per acre.

17. A man paid $100 for 1870 sq. yd. of land. How much was that per acre?

18. A man is to receive wages at the rate of $9.50 per week of 7 days. What will be the amount of his wages from 1st May to 1st December, both inclusive?

19. If $\frac{7}{20}$ of 3 shares in a company be worth $26.25, find the value of 15 shares.

20. Find the duty @ 2⅞ ct. a lb. net on 15 casks, each weighing 780 lb. gross, deducting $\frac{5}{56}$ of the gross weight as tare.

21. Wheat is quoted on Monday @ 91⅛ ct., on Tuesday @ 91⅜ ct., on Wednesday @ 91½ ct., on Thursday @ 91¼ ct., on Friday @ 91⅛ ct., on Saturday @ 89⅞ ct. What was the average price for the week?

22. On four successive days the barometer stood at $29\frac{1}{10}$ in., on the fifth day at $30\frac{3}{100}$ in., the following day at $30\frac{17}{100}$ in., and on the seventh day at 31 in. What was the weekly average?

23. A man took 7501 steps in walking $3\frac{7}{320}$ mi. What was the average length of his step?

24. A man whose steps average 2½ ft. in length walked 9 mi. in 2¼ hr. How many steps did he make on an average per minute?

25. In emptying a cistern a tap discharged an average of 24¼ gal per min. for the first 6⅓ min., an average of 19⅜ gal. per min. for the next 5½ min., an average of 13¾ gal. per min. for the next 3⅕ min., and a total of 29⅜ gal. in the next 7½ min., at the end of which time the cistern was empty. How many gal. did the cistern at first contain, and what was the average rate of discharge per min. for the whole time?

26. I bought 50 yd. of calico, part at 13 ct. a yd. and the remainder at 18 ct. a yd., and paid $7.62 for the whole. How many yards did I buy at each price?

27. How much tea costing 54 ct. a lb. must be mixed with 18 lb. costing 45 ct. a lb. in order that if the whole be sold at 60 ct. a lb. there may be a gain of $\frac{1}{5}$ of the cost of the whole?

28. On four consecutive days a train arrived at a certain station 15 min., 10 min. 30 sec., 11 min. 47 sec., and 13 min. 23 sec., respectively, past 10 a.m. If the train was on an average $3\frac{1}{4}$ min. late on those four days, at what time was it due at that station?

29. A Grand Trunk train left Montreal at 11.55 p.m. on Tuesday and arrived in Chicago at 7.25 a.m. on the following Thursday, having travelled a distance of 831 miles. Find the average speed of the train, Chicago time being one hour later than Montreal time. If 5 hr. 10 min. were lost in stoppages, find the average speed of running of the train.

30. Starting at 8.20 a.m., I find I have walked $4\frac{1}{4}$ mi. by 9.30 a.m. I then slacken my pace and walk $7\frac{1}{2}$ mi. farther by 12.05 p.m. What was the average rate in miles per hour at which I walked at first, and what the rate afterwards?

31. Divide $7\frac{1}{2}$ lb. of tea into two parcels one of which shall be $1\frac{3}{4}$ lb. heavier than the other?

32. Divide a string $5\frac{1}{2}$ yd. long into three pieces such that the first shall be $1\frac{1}{8}$ yd. shorter than the second, but $\frac{3}{4}$ yd. longer than the third.

33. Divide 3 yd. of tape into three parts so that the first shall be $\frac{3}{5}$ of the length of the second, and the second $\frac{10}{11}$ of the length of the third.

34. Divide 100 A. among A, B, C and D in such proportion that A shall have $2\frac{2}{3}$ times as much land as B, and C shall have $1\frac{3}{8}$ times as much as B, and D shall have $\frac{1}{2}$ as much as A, B and C together.

35. At an election one candidate polled 39 votes more than $\frac{5}{6}$ of the number polled for the other, the total number of votes cast being 1247. Find the number of votes cast for each candidate.

36. Divide \$79 among 8 men and 10 boys, giving each man \$2.07$\frac{1}{2}$ more than three-quarters of the amount given to each boy.

37. Divide $74\frac{1}{4}$ bu. of wheat between A and B so that if A give $\frac{1}{10}$ of his share to B they shall have equal quantities.

38. A man who had three sons, aged respectively 18, 12 and 10 years left his estate to be divided among them in proportion to their ages. What fraction of the estate is each to receive?

39. Annie is 12 yr. 4 m. and James is 15 yr. 5 m. old. Divide \$9 between them so that Annie shall receive 50 ct. more than she would receive were the money divided in proportion to their ages.

40. *A* and *B*, who were $22\frac{1}{2}$ mi. apart, commenced at the same moment to walk towards each other, *A* walking $1\frac{3}{11}$ mi. per hour faster than *B*. They met in 3 hr. 18 min. What were their respective rates of walking?

41. Three townships have to raise among them the sum of \$7450, each township to raise a part of this amount in proportion to its assessment. If the assessments are \$1,745,680, \$2,385,000 and \$4,763,540 respectively, find to the nearest cent the amount to be raised by each township.

42. Find the cost of the stair-carpet @ \$1.35 a yd. for a flight of 23 steps of $11\frac{1}{4}$ in. run $6\frac{1}{2}$ in. riser, allowing $1\frac{1}{4}$ yd. extra at top and $2\frac{1}{4}$ yd. extra for a turn in the stairs. (Reckon to nearest eighth of a yard.)

43. A map is drawn to the scale of 36 mi. to the inch. Find the total length of a railroad whose several parts measure on the map $3\frac{3}{8}$ in., $4\frac{5}{8}$ in., $2\frac{1}{8}$ in., $1\frac{7}{16}$, in., $1\frac{3}{16}$ in. respectively.

44. Of two rectangles of the same area, one is 7′ 6″ long by 6′ $3\frac{1}{4}$″ wide, and the other is $10\frac{1}{2}$″ wide. Find its length.

45. Find the area of a rectangle whose perimeter is 250 yd., and whose length exceeds its breadth by $25\frac{1}{4}$ yd.

46. Find the distance travelled in ploughing $6\frac{1}{4}$ A. of land, the furrow averaging 9 in. wide.

47. How long would it take to plow 7 A. 2904 sq. yd., the horses travelling $2\frac{1}{4}$ mi. per hr. and the furrow averaging $9\frac{1}{2}$ in. wide?

48. A field [343 yd. × $229\frac{1}{2}$ yd.] yielded 310 bu. of wheat. How many bu. was that per acre?

49. The scale of a certain map is 40 mi. to the inch. Find the area represented by a rectangle on the map $2\frac{5}{8}$ in. long by $1\frac{1}{2}\frac{1}{2}$ in. wide.

50. Find the number of sq. yd. in the total surface of a rectangular block of stone 7′ $3\frac{1}{2}$″ × 2′ 8″ × $6\frac{1}{2}$″.

51. A cube of cheese $3\frac{3}{4}$ in. on the edge was cut into cubes $\frac{3}{8}$ in. on the edge. How many of these were there?

52. How many cubic feet of plaster would be required to plaster a rectangular ceiling 18′ 8″ by 14′ 4″, the plaster to be $\frac{3}{8}$ in. thick?

Find the value of—

53. 29,650 ft. of lumber @ \$16.75 per M.

54. 1,270 boards 15′ × 13″ × $1\frac{1}{4}$″ @ \$18.50 per M.

55. 12,375 planks 12′ × 9″ × $2\frac{1}{2}$″ @ \$17.75 "

56. 1,750 scantlings 16′ × 7″ × $3\frac{1}{2}$″ @ \$23.75 "

57. 12,750 boards 16′ × 5″ × $\frac{3}{4}$″ @ \$31.50 "

58. A pile of cordwood $5\frac{1}{2}$′ × $26\frac{1}{2}$′ @ \$3.75 a cord.

59. If from one end of a stick of square timber $21' \times 17\frac{1}{2}'' \times 13\frac{1}{2}''$ there be cut off $7\frac{1}{2}$ cu. ft., find the length of the stick remaining.

60. A pile of bricks 12 bricks long by 20 wide by 25 high measured 8 ft. 2 in. by 6 ft. 11 in. by 5 ft. 6 in. Find the volume and the dimensions of an average-sized brick.

61. How many tons of earth must be removed to add 15″ to the depth of a canal 7 mi. 425 yd. long and averaging 25 ft. wide, if a cubic yard of earth weigh 2956 lb.?

62. Find the weight of the sleepers for $37\frac{1}{4}$ mi. of railway if the sleepers average 7′ 9″ long, 10″ broad and 8″ thick, and are laid 2 ft. 3 in. from centre to centre, and weigh $32\frac{7}{8}$ lb. per cu. ft.

63. Find the average thickness of a slate $8'' \times 10''$ which weighs 25 oz., if a cubic foot of slate weigh 180 lb.

64. How many gallons of water will pass under a bridge every 10 min., if the stream be 28′ 8″ wide, average 3′ 6″ deep, and flow at the rate of $3\frac{1}{4}$ mi. per hour?

65. Into a rectangular cistern whose floor measures 6′ $4\frac{1}{2}''$ by 4′ $9\frac{1}{2}''$ water is flowing at the rate of 500 gal. per hour. How long will it take to fill the cistern to a depth of 3′ $10\frac{1}{2}''$?

66. Find the cost @ $18.65 per M. of the lumber for a board fence 5 boards high to enclose a rectangular field 65 rd. × 36 rd., the lumber to be inch stuff 7 in. wide.

67. A lidless rectangular box, whose outside measurements are 4′ $3\frac{1}{2}''$ long × 2′ $7\frac{3}{4}''$ wide × 2′ deep, is made of wood $1\frac{1}{8}''$ thick. Find its contents in cubic feet.

68. A boy spent $\frac{3}{5}$ of his money and then had $1.20 left. How much had he at first?

69. A, working on piece-work, can do only $\frac{8}{11}$ as much work as B, and so earns 55 ct. per day less than B. How much does each earn per week?

70. A man sold $\frac{1}{3}$ of his farm and then $\frac{2}{5}$ of the remainder. How much of the farm did he sell? If he received $1210 from both sales, at that rate what was the value of his farm?

71. A man paid $\frac{1}{2}$ of his money to B, $\frac{1}{3}$ of it to C, and $\frac{3}{5}$ of the remainder to D, and had 15 ct. left. How much had he at first?

72. A, B and C have to do a piece of work. A does $\frac{5}{11}$ and goes away; B does $\frac{2}{3}$ of the remainder, and then C finishes it. What fraction of the work is done by C? If $16.50 be paid for the whole work, how much should each receive?

73. If $\frac{5}{8}$ of 2 lb. of sugar cost as much as $2\frac{1}{2}$ lb. of rice, and if $3\frac{3}{4}$ lb. of rice cost 15 ct., what is the price of the sugar per pound?

74. By selling a house for $3990 I lost $\frac{1}{8}$ of its cost. For what amount should I have sold it to gain $\frac{1}{8}$ of its cost?

75. A can walk 4 mi. while B walks 5 mi., and B can walk 6 mi. while C walks 5 mi. Compare A's rate of walking with C's rate.

76. How far will a train travel in 1 hr. 35 min. at the rate of $9\frac{5}{7}$ mi. in $14\frac{1}{2}$ min.?

77. A watch is set right at 10.25 p.m., and it gains $3\frac{1}{3}$ sec. every hour. At what o'clock will it have gained exactly $\frac{3}{8}$ of an hour, and what time will the watch then indicate?

78. A boat's crew can row at the rate of $9\frac{1}{4}$ mi. an hour in still water. At what rate could they row, 1st up, 2nd down, a stream running at the rate of $2\frac{5}{8}$ mi. an hour?

79. An oarsman rowed $3\frac{1}{4}$ mi. down stream in 20 min., and back again up stream in 36 min. Find his rate per hour each way, his rate in still water, and the rate of the stream.

80. A train 44 yd. in length is running at the rate of 24 mi. per hour. How long will it take in passing a man—1st, if he stand still while the train passes; 2nd, if he walk at the rate of 3 mi. per hour in the direction the train is moving; 3rd, if he walk at the rate of 3 mi. per hour in the opposite direction?

81. A train 220 ft. in length is running at the rate of 25 mi. per hour. How long will it take to pass another train 330 ft. long if the second train be (i), standing on a parallel track; (ii), moving in the opposite direction at the rate of 15 mi. per hour; (iii), moving in the same direction at the rate of 15 mi. per hour?

82. Two men who are $12\frac{1}{4}$ mi. apart start at the same moment to travel towards each other, one walking at the rate of $3\frac{1}{4}$ mi. per hr., the other driving at the rate of $10\frac{1}{2}$ mi. per hr. How long after starting will it be till they meet, and how far will the first man have walked?

83. A cistern which holds 200 gal. can be filled by two taps, of which one supplies $\frac{4}{125}$ gal. per sec., the other $1\frac{3}{5}$ qt. per sec. If the first tap be turned on for 10 min. and afterwards both run together, in what length of time from the moment of opening the second tap will the cistern be filled?

84. A can do a certain piece of work in 12 da., and B can do it in 15 da. How much of the work can each do per day? How much can both together do in a day? How long would it take the two, working together, to do the work?

85. If 5 men *or* 16 boys can do a certain piece of work in 11 hr., in what time could 3 men *and* 48 boys do the work?

86. A can do a piece of work in 10 da.; B can do it in 12 days. A works at it alone for $4\frac{1}{2}$ da., and is then joined by B. In how many days will the two, working together, finish the job?

87. G can do as much work in 4 days as H can do in 5 days, or as much in 5 days as M can do in 9 days. The three undertake a contract and G and H work together on it for 18 days, then M takes G's place and H and M work together on it for 26 days and thus finish the contract. How long would it have taken G working all the time alone to have executed the contract?

88. A can do a certain piece of work in $\frac{1}{2}$ da., B can do it in $\frac{3}{4}$ da., and C can do it in $1\frac{1}{4}$ da. In what time could all three working together do it?

89. A and B start at the same moment to run in the same direction round a circular track, A making 8 rounds to B's 5. Where will A overtake B the first time? the second time? the third time? How many rounds will each have made on each occasion?

90. The hour and the minute hands of a clock are together at 12 o'clock. At what times will they be together again? At what times will they point in opposite directions? At what times will they be 15 min. apart?

91. At what times will the minute-hand be half as many minute-spaces ahead of the hour-hand as the hour-hand marks hours?

92. A grocer drew off 4 gal. from a full barrel of vinegar and filled the barrel up with water. Next day he drew off 4 gal. of the mixture and then filled up the barrel with water. On the third day he drew off 4 gal. of the mixture and filled up the barrel with water. If the barrel held just 32 gallons, how many gallons of the vinegar originally contained in the barrel remained in it after the third drawing off?

93. There are three vessels—A of 1 gal., B of 2 gal. and C of 5 gal. capacity. A is empty, B is full of water, and C is full of vinegar. A is filled from B, B is replenished from C, and A is then emptied into C. This is done three times, the water and vinegar being thoroughly mingled at every mixture. How much water is there then in C?

94. The area of Canada is about 14 times that of the Austro-Hungarian Empire, $14\frac{7}{10}$ times that of Spain and Portugal, $16\frac{2}{3}$ times that of the German Empire, $16\frac{4}{5}$ times that of France, 30 times that of Italy, 132 times that of Holland and Belgium and 216 times that of Switzerland. Show that Canada is more than thrice as large as all these states together.

CHAPTER VII.

DECIMALS.

I. NOTATION AND NUMERATION.

In the ordinary or Arabic notation a figure standing immediately to the right of another denotes so many units each ten times less than the unit of that other. Thus in 325 the unit of the 3 is a hundred; that of the 2 is ten, which is *a tenth of a hundred;* and that of the 5 is one, which is *a tenth of ten.* By continuing this system beyond the ones, a figure immediately to the right of the *ones* would denote *tenths;* the next figure to the right would denote tenths of tenths, or *hundredths;* the next figure to the right would denote tenths of hundredths, or *thousandths*, and so on. In the case of numbers thus containing figures denoting units less than ones, the figure which denotes ones is indicated by a dot (·) called the **Decimal Point** placed between it and the figure denoting tenths. Thus 4 hundreds, 2 tens, 7 ones, 8 tenths, 5 hundredths and six thousandths would be written 427·856.

The units denoted by figures to the right of the decimal point are called **Decimal Units.** A number containing decimal units is called a decimal number, or, briefly, a **Decimal**; and the part to the right of the decimal point is called the **Decimal Part,**—the part to the left is integral.

The **Order of a Unit** is its rank as determined by the number of times the prime unit must be multiplied by 10 or divided by 10, as the case may be, to produce one of that unit. Thus, tens are of the first integral order for $10 = 1 \times 10$; hundreds are of the second integral order, for $100 = 1 \times 10 \times 10$; thousands are of the third integral order, for $1000 = 1 \times 10 \times 10 \times 10$, and so on;

tenths are of the first decimal order, for $0{\cdot}1 = 1 \div 10$; hundredths are of the second decimal order, for $0{\cdot}01 = 1 \div 10 \div 10$; thousandths are of the third decimal order, for $0{\cdot}001 = 1 \div 10 \div 10 \div 10$, and so on. The prime units or ONES are of the zeroth order. The greater the number of multiplications, or the less the number of divisions by 10, the higher the order; the fewer the multiplications, or the more numerous the divisions, the lower the order. Thus hundreds are of higher order than tens, but of lower order than thousands; while hundredths are of higher order than thousandths, but of lower order than tenths, or ones, or tens.

The number 324·657 represents 324 and 6 tenths, 5 hundredths and 7 thousandths, and might be so read; but since 1 of any order is equal to 10 of the next lower order, 6 tenths and 5 hundredths is 65 hundredths, and 65 hundredths and 7 thousandths is 657 thousandths. 324·657 is therefore read 324 *and* 657 thousandths. Similarly 4,023,148·478,602,5 is read 4 million 23 thousand 148 *and* 478 thousandths 602 millionths 5 tenths of millionths.

Another way of reading decimal numbers, and one that is very convenient in practice, is to read the integral parts in the usual way, then to say "point" (or "decimal"), and then to name in succession from the left the figures of the decimal part. Thus 127.00435 is read "127, *point* 0, 0, 4, 3, 5.".

EXERCISE LXII.

Read—

1. 7·56.	*4.* 0·2304.	*7.* 1·0001.	*10.* 1325000·625.
2. 0·756.	*5.* 2·304.	*8.* 1000·1.	*11.* 13·25000625.
3. 75·6.	*6.* 0·002304.	*9.* 0·010001.	*12.* 132500·0625.

Write in Arabic notation—

13. Seven thousand three hundred and forty-nine *and* four hundred and six thousandths.

14. One million *and* seventy thousandths six millionths.

15. One thousandth and one hundredth of a thousandth.

16. Three thousand and nine *and* two hundred and seventy thousandths 8 millionths and one tenth of a millionth.

17. Name the orders of the several figures in questions 6, 10 and 11 above.

II. ADDITION AND SUBTRACTION OF DECIMALS.

Decimals are added and subtracted exactly as integers are.

In arranging numbers for addition or for subtraction, all figures denoting units of the same order, and only these, must stand in the same vertical column. To secure this, *write the given numbers so that their decimal points shall be in a vertical column. The decimal point of the sum or the difference will be under the other decimal points.*

EXERCISE LXIII.

Add together—

1. 37·645, 283·039, 5847·036, 86·453 and 3·768.
2. 45983·7, 4·59837, 45·9837, 459837 and 0·459837.
3. 0·00876, 1·08972, 1000, 0·0009 and 900·009.
4. 36400, 0·00364, 287·082, 5·78936 and 307·125.

Subtract—

5. 97·46 from 368·24.
6. 109·87 from 193·857.
7. 0·777 from 7.
8. 0·9999 from 10.

Find the value of—

9. 37·5 + 48·26 + 0·00831 − 85·759.
10. 2·02 − 0·0909 − 1·9009 + 19·009 − 9·029209.

11. John had $7·38 more than James. John spent $29·13; James spent $19·45. How much had James more than John then?

12. A man sold ·375 of his farm. How much of it had he left?

13. Three men did a certain piece of work. The first did 0·37 of it and the second did 0·33 of it. How much of it did the third man do?

14. A farmer had 23·478 A. in one field, 29·38 A. in a second field, 18·076 A. in a third field, 0·875 A. occupied by barns and as barnyard, and 1·305 A. taken up with house, garden and orchard. The rest of his farm, which consisted of 100 A. in all, was in woodland. How many acres of woodland had he?

III. MULTIPLICATION OF DECIMALS.

Decimal numbers are multiplied together exactly as integral numbers are. The reasoning which proves that in multiplying by any number of *integral* units the order of the units of the product is *higher* than the order of the units of the multiplicand by the order of the units of the multiplier, also proves that in multiplying by any number of *decimal* units the order of the units of the product is *lower* than the order of the units of the multiplicand by the decimal order of the units of the multiplier. Thus multiplying by hundreds raises hundredths to ones, tenths to tens, ones to hundreds, tens to thousands, and so on; multiplying by hundredths lowers tens to tenths, ones to hundredths, tenths to thousandths, and so on.

Examples.—Multiply 237·6 by two hundred and one, and also by one and two hundredths.

(1)	(2)
237·6	237·6
201	1·02
237·6	237·6
4752	4·7 52
47757·6	242·3 52

Hence to multiply two decimal numbers together,

Write the multiplier under the multiplicand so that the ones' figure of the multiplier may be under the right-hand figure of the multiplicand.

Multiply by each digit of the multiplier in any convenient order of succession, and write each partial product so that the right-hand figure shall be in the same vertical column as the digit of the multiplier which produced it.

Add all the partial products together; their sum will be the required product.

The decimal points of the multiplicand, the partial products and the total product will all be in the same vertical column.

Examples—

(1)	(2)	(3)
4·725	4·725	4·725
123	1·23	·00123
14·175	·141 75	·000 14175
94·50	.945 0	·000 9450
472·5	4.725	·004 725
581·175	5·811 75	·005 81175

It is usual to omit from the partial products their decimal points and the noughts on their left. When this is done, the rule for the multiplication of factors containing decimals may be stated—

Multiply the factors together as if they were integral, and from the right hand of the product mark off for decimals as many figures as there are decimal places in all the factors taken together. Should the number of figures in the product be less than the number of figures to be marked off, supply the deficiency by writing noughts on the left of the product.

The value of a decimal is not changed by writing noughts to the right of a decimal part or by removing such noughts; for the presence or the absence of these noughts has no effect on the order of the units of the other figures, and therefore has no effect on their value, and the noughts themselves have no value. Thus $7\cdot30 = 7\cdot3 = 7\cdot300$.

EXERCISE LXIV.

Multiply—

1. 37·45 by 10, 100, 1000, 100000, 0·1, 0·01, 0·001, 0·00001.
2. 0·0072 by 100, 0·01, 10000, 0·0001, 1000000, 0·1.
3. 10 by 0·04, 0·006, 7000, 0·00002, 20·00.

[*When a decimal number contains no integral part this may be indicated by writing a nought in the ones' place, as is done in the three preceding problems, but as this nought is not absolutely necessary, it is customary to omit it.*]

Find the value of—

4. $783\cdot46 \times 7$.	8. $\cdot0476 \times 42$.	12. $1\cdot476 \times \cdot0003$.
5. $783\cdot46 \times 70$.	9. $\cdot0476 \times 4\cdot2$.	13. $\cdot00039 \times \cdot39$.
6. $783\cdot46 \times \cdot7$.	10. $\cdot0476 \times \cdot42$.	14. $\cdot079 \times 300 \times \cdot03$.
7. $783\cdot46 \times 700\cdot07$.	11. $\cdot0476 \times 4\cdot02$.	15. $\cdot004 \times \cdot005 \times \cdot5$.

IV. DIVISION OF DECIMALS.

Case I.—When the divisor is integral,

Divide as if divisor and dividend were both integral; but as soon as the division has been carried to the end of the integral part of the dividend, place a decimal point after the figures thus far obtained in the quotient, and then continue the division.

If the quotient is required to be correct to more decimal places than there are in the dividend, annex noughts to the right of the dividend until there are the requisite number of decimal places. Add 1 to the last figure of the quotient if the remainder is a half or more than a half of the divisor—that is, if the next figure of the quotient would be 5 or more than 5 were the division carried one place farther.

Examples—

(1) 74·358 ÷ 9.

9)74·358

8·262

(2) 135·7 ÷ 7 to 3 decimal places.

7)135·700

19·386 –

Case II.—When the divisor is a decimal,

If necessary, annex decimal noughts to the dividend until it contains at least as many decimal places as the divisor does;

Remove the decimal point to the right of the divisor and remove it an equal number of places to the right in the dividend;

Then divide as in Case I.

Removing the decimal points to the right multiplies both divisor and dividend by 10 as many times as the point is removed places. Divisor and dividend are thus multiplied both by the same number; the quotient will therefore not be affected.

Examples—

(1) 72·45 ÷ ·9.

9)724·5

80·5

(2) 46·2 ÷ ·08.

08)4620·0

577·5

(3) ·001 ÷ ·0003.

0003)0010

3·333 +

In example 3 the decimal noughts are not actually written down, but the work is continued as if they were there.

Removing the decimal point of any number 1, 2, 3,....places to the right multiplies *the number by 10, 100, 1000,....; removing the decimal point 1, 2, 3,.... places to the left* divides *the number by 10, 100, 1000,.....* For by removing the decimal point *one* place to the right the value of the unit of each figure composing the number is increased ten-fold, and therefore the whole—that is, the number—is increased ten-fold. Removing the decimal point *two* places to the right increases the value of the number ten times ten-fold, or a hundred-fold. In like manner the other cases may be proved.

EXERCISE LXV.

Divide—

1. 438·976 by 7, 8, 9, 11, 79, 474.
2. 256·43 by 4, 6, 7, 17, 25, to 4 decimal places each.
3. 40·04 by 10, 100, 1000, 7, 70, 700, 110, 13000, 1300.
4. 72.09 by 10, 0·1, 100, 0·01, 1000, 0·001, 0·00009, 0·0089.

Find, correct to 4 decimal places, the value of—

5. 1·075 ÷ ·125.
6. ·004 ÷ ·5.
7. ·04 ÷ ·005.
8. 40 ÷ ·0005.
9. 7·29 ÷ 036.
10. 547 ÷ ·007.
11. ·8 ÷ ·0064.
12. 6 ÷ ·000725.
13. 11·02 ÷ ·0032.
14. 8·0018 ÷ 900.
15. ·006 ÷ 70.
16. ·008 ÷ 8·8.

EXERCISE LXVI.

1. One hundred and twenty steps, each 5·875 in. high lead from the foot to the top of a tower. What is the height of the tower?

2. The side of a square plot of ground measures 13·3375 yd. What is its area?

3. How many cubic feet of water will fill to the depth of 6·75 ft. a rectangular tank 25·475 ft. long by 15·64 ft. wide?

4. The average annual death-rate in a city of 64,000 inhabitants is 23·5625 per 1000. Find the total number of deaths in 7 years.

5. In every 1000 parts by weight turnips contain 905 parts water. How many gallons of water are there in 1000 bushels of turnips?

6. In every 1000 parts by weight rice contains 741 parts of starch, and potatoes contain 155 parts. How much starch would be contained in 1 lb. of each? How many pounds of rice would contain as much starch as 100 bu. of potatoes?

V. INTERCONVERSION OF DECIMALS AND FRACTIONS.

To express a decimal as a mixed number or a fraction,

Write the decimal part for numerator, omitting the decimal point, and for denominator write 1 followed by as many noughts as there are decimal places in the given number. Reduce the resulting fraction to lowest terms.

Ex. 1.—$2{\cdot}5 = 2\frac{5}{10} = 2\frac{1}{2}$.

Ex. 2.—$13{\cdot}0375 = 13\frac{375}{10000} = 13\frac{3}{80}$.

Ex. 3.—$\cdot00064 = \frac{64}{100000} = \frac{2}{3125}$.

A fraction whose denominator is 1 followed by one or more noughts is called a **Decimal Fraction.**

EXERCISE LXVII.

Express as fractions in their lowest terms—

1. ·25.	*3.* 1·476.	*5.* ·024.	*7.* 70·64.	*9.* 3·62500.
2. 1·75	*4.* ·1476.	*6.* ·0024.	*8.* 7·064.	*10.* 3·00625.

To express a fraction as a decimal number correct to a given number of decimal places,

Annex to the numerator a decimal nought for each decimal place required and divide by the denominator.

Increase the last figure of the quotient by 1, if the next figure would have been 5 or upwards had the division been continued.

EXERCISE LXVIII.

Express as decimals correct to 5 decimal places—

1. $\frac{7}{10}$.	*3.* $1\frac{5}{8}$.	*5.* $\frac{485}{512}$.	*7.* $\frac{22}{7}$.	*9.* $\frac{5}{13}$.
2. $\frac{13}{1000}$.	*4.* $4\frac{77}{125}$.	*6.* $\frac{693}{640}$.	*8.* $\frac{355}{113}$.	*10.* $\frac{19}{43}$.

11. Arrange $\frac{18}{29}$, $\frac{23}{37}$, $\frac{64}{103}$, $\frac{343}{552}$ in order of magnitude

Find, correct to 4 decimal places, the value of—

12. $\frac{3}{7} + \frac{7}{8} + \frac{8}{11}$.	*15.* $13\frac{3}{8} + 5\frac{2}{5} - 8\frac{33}{55}$.
13. $1\frac{5}{12} + 2\frac{7}{20} + 4\frac{13}{28} + \frac{11}{30}$.	*16.* $4\frac{17}{46} + 3\frac{11}{81} - 7\frac{313}{623}$.
14. $23\frac{19}{20} + 3\frac{5}{12} + 11\frac{13}{27}$.	*17.* $\frac{789}{1001} + \frac{884}{999} - \frac{455}{926}\frac{83}{87}$.

VI. DENOMINATE DECIMALS.

EXERCISE LXIX.

Express as a compound denominate number—

1. 3.4758 T. *2.* 4·2625 yd. *3.* 29·530875 da.

4. How many seconds are there in ·001168 da. ?

5. Reduce ·000875 A. to sq. in.

6. Express 833 yd. 2 ft. 9 in. as a decimal of a mile, correct to 4 decimal places.

7. Express 1 da. 18 hr. 28 min. 35·945 sec. as a decimal of a day, correct to 6 decimal places.

8. Express 4 ch. 45 l. as a decimal of a chain.

9. Express 12 A. 3 sq. ch. 7500 sq. l. as a decimal of an acre.

10. Find the area expressed in acres of a rectangular field 17 ch. 50. l. by 10 ch. 20 l.

11. If a cubic foot of water weigh 62·356 lb., how many cubic inches of water will weigh 10 lb.?

12. How many gallons will fill a rectangular tank measuring 18′ 6″ by 13′ 5″ by 5′ 8″? (A gallon = 277.118 cu. in.)

13. What will be the weight of the air in a rectangular room measuring 34 ft. 3 in. by 26 ft. 9 in. by 13 ft. 6 in. if every cubic foot of the air weigh 536·268 grains?

14. The tropical year consists of 365·24216 mean solar days; express this as a compound number of days, hours, etc.

15. In 400 successive calendar years there are 303 years of 365 days each and 97 years of 366 days each. Find the average length of the calendar year and show that it exceeds the tropical year by about one day in 3600 years.

(A metre = 39·370432 inches.)

16. How many metres are there in a mile?

17. How many yards are there in 1000 metres?

18. How many miles are there in 40,000,000 metres?

19. How many square inches are there in a square metre?

20. How many square yards are there in a square measuring 10 metres on each edge?

21. How many square metres are there in an acre?

22. How many cubic inches are there in a litre, which is the thousandth part of a cubic metre?

23. How many gallons are there in 1000 litres, *i.e.*, in a cubic metre?

CHAPTER VIII.

APPLICATIONS OF DECIMALS.

I. PERCENTAGES.

The phrase **per cent.**, a shortened form of the Latin *per-centum*, is equivalent to the English word **hundredths**. Thus, 3 per cent. of any quantity is 3 hundredths of it; $12\frac{1}{2}$ per cent. is $12\frac{1}{2}$ hundredths, and 135 per cent. is 135 hundredths. The symbol % is frequently employed to denote the words *per cent.*, and may be read either *per cent.* or hundredths. Hence $3\% = \cdot 03$, $20\% = \cdot 20$, $12\frac{1}{2}\% = \cdot 125$, 8% of $450 = \cdot 08$ of $450 = 36$, $3\frac{1}{3}\%$ of $165 = \cdot 03\frac{1}{3}$ of $165 = 5 \cdot 5$.

EXERCISE LXX.

Read the following rates and write them decimally :—

1. 5%. *2.* $7\frac{1}{2}\%$. *3.* $33\frac{1}{3}\%$. *4.* 150%. *5.* $\frac{1}{2}\%$.

Write the following decimals as percentages :—

6. ·07. *7.* ·70. *8.* ·375. *9.* 2·25. *10.* ·0075.

How much is—

11. $3\frac{3}{4}\%$ of \$700? *13.* 125% of 120 yd.? *15.* $102\frac{1}{4}\%$ of \$12.50?

12. 10% of \$225? *14.* $12\frac{1}{2}\%$ of 44 lb.? *16.* $\frac{3}{8}\%$ of \$75.80?

What rate per cent is—

17. \$3 per \$50? *18.* 8 lb. per 250 lb.? *19.* 9 in. per 100 yd.?

What percentage of—

20. \$150 is \$6? *21.* 489 gal. is 60 gal.? *22.* 750 A. is $18\frac{3}{4}$ A.?

Express the following percentages as fractions in their lowest terms :—

23. 25%. *24.* 20%. *25.* $12\frac{1}{2}\%$. *26.* $33\frac{1}{3}\%$. *27.* $162\frac{1}{2}\%$.

28. Increase \$225 by 8% of itself.

29. Decrease \$360 by 6% of itself.

30. Decrease 1250 gal. by 8% of itself, and then increase the remainder by 8% of *itself*.

31. A farmer who had 80 sheep sold 20% of them. How many did he sell?

32. A man bought a house for \$1760. For how much must he rent it to obtain $12\frac{1}{2}$% per annum on this price?

33. A teacher spent on books \$47.25, which sum was 7% of his salary. Find the amount of his salary.

34. The average attendance of pupils at a certain school was 55, which was $62\frac{1}{2}$% of the number of pupils enrolled. Find the number of pupils enrolled.

35. Willie Smith gained $8\frac{1}{4}$ lb. in weight in twelve months; this was an increase of $7\frac{1}{2}$% of his weight at the beginning of the 12 months. What was his weight at the beginning of the 12 months?

36. A house worth \$2750 rents for \$320 a year. For what percentage of its value does it rent?

37. The total population of Canada in 1881 was 4,324,810. Of this number 609,318 were not born in Canada. What percentage of the population was born outside of Canada?

38. In 1884 the values of the several classes of exports from Canada of Canadian production were:—Produce of the mine, \$3,247,092; of the fisheries, \$8,591,654; of the forest, \$25,811,157; animals and their produce, \$22,946,108; agricultural products, \$12,397,843; manufactures, \$3,577,535; miscellaneous articles, \$560,690. Find the percentage which these separate values form of their total value.

39. A man spent 85% of his income of \$850. How much had he left?

40. A man who was receiving \$8·40 a week had his wages increased by 8%. Find the amount of his wages per week after the increase.

41. A man whose wages had been increased 10% was then in receipt of \$8·14 per week. How much did he receive per week before the increase?

42. A house was sold for \$3451, which was 15% less than it had cost to build. Find how much it had cost.

43. A man's wages were decreased from \$7·80 a week to \$7·20 a week. Find the rate % of decrease.

44. From a barrel of 36 gal. of oil 8 gal. were drawn off. What percentage of the original quantity remained?

II. APPLICATIONS OF PERCENTAGE.

PROFIT AND LOSS.

The **Prime Cost** of merchandise or other property is the net sum paid by the purchaser thereof to the seller thereof.

The **Gross Cost** of merchandise or other property is the sum of the prime cost, all charges for purchasing, and all expenses for freight, storage, handling, and such like.

Profit is the amount by which the selling price exceeds the cost price. *Net Profit* or *Gain* is the amount by which the selling price exceeds the gross cost.

The **Rate of Profit** is usually expressed as a percentage of the prime cost.

Thus, if goods costing \$5 are sold for \$6·20,
the PROFIT is $\$6{\cdot}20 - \$5{\cdot}00 = \$1{\cdot}20$,
and the RATE *of profit* is $\frac{\$1{\cdot}20}{\$5{\cdot}00} = {\cdot}24 = 24\%$.

Loss is the amount by which the selling price falls short of the cost price. *Net loss* is the amount by which the selling price falls short of the gross cost.

The **Rate of Loss** is usually expressed as a percentage of the prime cost.

Thus, if goods costing \$12 are sold for \$9·60,
the LOSS is $\$12{\cdot}00 - \$9{\cdot}60 = \$2.40$,
and the RATE *of loss* is $\frac{\$2{\cdot}40}{\$12{\cdot}00} = {\cdot}20 = 20\%$.

EXERCISE LXXI.

Find the profit or the loss and the rate of profit or of loss, given :—

	Cost.	*Selling Price.*		*Cost.*	*Selling Price.*
1.	\$12.	\$15.	*4.*	\$450.	\$512·50.
2.	\$150.	\$180.	*5.*	\$3·75.	\$2·00.
3.	\$225.	\$198.	*6.*	\$500.	\$500·50.

Find the profit or the loss and the selling price, given:—

	Cost.	*Rate of Profit.*		*Cost.*	*Rate of Loss.*
7.	\$150.	6%.	*10.*	\$42·50.	10%.
8.	\$225.	5%.	*11.*	\$250.	$1\frac{1}{2}$%.
9.	\$137·50.	36%.	*12.*	\$1600.	$3\frac{3}{8}$%.

13. What will be the rate of selling price if the rate of profit be 6%? 11%? 20%? $7\frac{1}{2}$%? $33\frac{1}{3}$%? 110%?

14. What will be the rate of selling price if the rate of loss be 4%? 7%? 10%? $7\frac{1}{2}$%? $33\frac{1}{3}$%? $3\frac{1}{3}$%?

Find the cost, given:—

	Selling Price.	*Rate of Profit.*		*Selling Price.*	*Rate of Loss.*
15.	\$17·60.	10%.	*18.*	\$14·40.	10%.
16.	\$38·00.	$33\frac{1}{3}$%.	*19.*	\$38·00.	$33\frac{1}{3}$%.
17.	\$3744.	125%.	*20.*	\$1094·50.	$\frac{1}{2}$%.

21. If a grocer were to sell at a profit of 15% tea which cost him 48 ct. a lb., how much would he receive for 85 lb., and how much of this would be profit?

22. Silk which cost \$2·40 a yd. is marked at 20% loss. Find the selling price and the rate of this selling price on the dollar of cost.

23. A merchant paid for freight and other expenses on certain stoves \$5 each over the cost price. He sold them for \$35 each, which was 40% advance on the cost price. Find his net gain and his rate of profit on the gross cost.

24. A man buys a bankrupt stock, which originally cost \$1860, paying therefor 65c. on the \$1 of original cost. How much does he pay for it?

25. A man buys at 55c. on the \$1 a bankrupt stock which cost per invoices \$2400, and sells it at an average of 95c. on the \$1. How much does he pay for it? How much does he sell it for? What is his rate of profit?

26. A man buys a bankrupt stock at 60c. on the \$1 per invoices, and sells it at an average of 5% advance on the invoice cost. Find his rate of gain.

27. A man buys at 68c. on the \$1 a bankrupt stock which cost per invoices \$5376. Half of it he sold at 5% above the invoice prices, a third of it he sold at 12% below the invoice prices, and the remainder he sold at half the invoice prices. Find his total gain and his rate of gain.

28. 20% of a cargo was lost; at what rate of profit must the remainder be sold if the investment is to yield 20% profit?

INSURANCE.

Insurance is a contract by which one party, *the insurer*, in consideration of a sum of money received from another party, *the insured*, engages to pay a stipulated sum on the happening of a particular event or undertakes to indemnify the insured or his representatives for loss or damage arising from certain specified causes, if sustained within a stated time.

The instrument or document setting forth the contract is termed an **Insurance Policy.**

The sum paid by the insured to the insurer is styled the **Premium.** It is generally a fixed percentage of the amount insured.

The **Term** of an insurance is the period for which the contract is made and the risk assumed.

The ordinary kinds of insurance are **Fire Insurance, Marine Insurance** and **Life Insurance.**

In **Fire Insurance,** the insurer undertakes to indemnify the insured up to a specified sum, for loss or damage that may occur to certain property described in the policy, if caused by fire, within a stated time, generally one, two or three years.

In **Marine Insurance,** the insurers contract to indemnify the insured up to a stipulated sum for any loss or damage that may occur to a certain ship, cargo or freight, any or all of them, by storms or other perils of navigation during a particular voyage or within a specified period not usually exceeding twelve months.

In *Life Insurance*, the insurer engages to pay on the death of the insured, a sum specified in the policy. In an *Endowment Policy*, the stipulated sum is payable to the insured if he should survive a specified number of years, but should he die before the expiration of the period named, the sum assured is to be paid to the representatives of the insured or to a person named in the policy.

Fire and life insurances are usually undertaken by companies or corporations organized to carry on such business ; marine insurance is undertaken both by companies and by private

persons. A marine insurance by private individuals is generally undertaken by several parties and each of them writes his name under or at the foot of the policy, and engages on his own account to indemnify the insured to the amount set opposite his name: on this account individual marine insurers are called *underwriters.*

In an ordinary fire policy, if the loss is only partial, the insurer undertakes to pay the full value of the property destroyed or the full amount of the depreciation of the property damaged, provided it does not exceed the sum covered by the insurance. In marine policies there is commonly an average clause which declares that the indemnity for a partial loss of property not insured to its full value will be the same part of the loss as the sum covered by the insurance is of the full value of the property.

If a property is insured in two or more companies or by two or more underwriters, the insurers are liable for the indemnity for a partial loss, in sums proportionate to the amounts of the risks severally assumed by them.

EXERCISE LXXII.

1. A factory valued at \$35,000 was insured for $\frac{3}{5}$ of its value, the rate of insurance being $\frac{5}{8}\%$ for one year. What was the amount of the premium?

2. A warehouse valued at \$62,500 was insured for $\frac{4}{5}$ of its value, the rate of insurance was $1\frac{1}{4}\%$ for three years, and the cost of the policy and the agent's expenses were \$2.50. What was the amount paid for the insurance?

3. What will be the cost of insuring a cargo of 24,000 bushels of wheat valued at \$1.05 per bushel, the insurance covering $\frac{4}{5}$ of the value of the cargo, the premium rate being $1\frac{1}{4}\%$ and the other expenses of the insurance being $2\frac{1}{2}\%$ of the premium?

4. A merchant's stock was insured for \$42,000, $\frac{1}{2}$ of this amount being at $\frac{7}{8}\%$, $\frac{3}{4}$ of the remainder at $\frac{3}{4}\%$ and the remainder at $\frac{5}{8}\%$. Find the total amount of premium paid.

5. A building and contents are insured as follows:— \$12,000 in the Imperial, \$8000 in the National and \$5,000 in the Lancashire Insurance Company. Were a loss to the extent of \$3500 to occur through fire, what portion of the loss should each company bear?

6. Merchandise valued at \$63,000 was insured in the Phœnix Insurance Co for \$15,000, in the North British and Mercantile Insurance Co. for \$12,000 and in the Norwich Union Fire Insurance Society for \$8000; if the merchandise is damaged by fire to the extent of \$10,500, how much of the damage should each company pay?

7. A merchant insured his stock for \$33,000 for one year at $\frac{7}{8}\%$. Six months thereafter the policy was cancelled at the request of the insured. Find the amount of premium returned, the short rate for six months being $\frac{5}{8}\%$.

8. A factory and the machinery therein is insured for \$65,000; $\frac{2}{5}$ of this sum is at $\frac{3}{4}\%$ premium and the remainder is at $\frac{7}{8}\%$. What is the average rate per cent. of premium paid on the whole?

9. A fire insurance company insured a building for \$60,000 at $\frac{7}{8}\%$ premium and reinsured one-half of the risk in another company at $\frac{5}{8}\%$ and one-third of the risk in a third company at $\frac{3}{4}\%$. What amount and what rate of premium did the company net on the remainder of their risk?

10. A steamboat worth \$60,000 is insured in three companies, in two to the amount of \$15,000 each and in the third to the amount of \$20,000. For what sum would each company be liable if the vessel were to sustain damage to the extent of \$6600?

11. A ship worth \$56,000 was insured for \$15,000 in one insurance company at $\frac{7}{8}\%$ premium and for \$32,000 in another company at $\frac{3}{4}\%$. The vessel received damage in a storm to the extent of \$7500. What amount had each company to pay to the owners of the vessel and by how much did each amount exceed the premium received by the company paying that amount?

12. A fire insurance company charged \$196·88 for insuring a house for \$17,500. What was the rate per cent.?

13. A merchant's stock was worth \$120,000; he insured it at $\frac{2}{3}$ its value paying \$700 premium. What was the rate per cent.? What was the rate in cents per \$100?

14. A shipment of goods is insured for \$7500 and \$18·75 is paid as premium. At that rate, what would be the amount of the premium on \$18,750?

15. The sum of \$285 was paid for the insurance at $\frac{3}{4}$ of its value of a ship worth \$50,000. What was the rate per cent. of premium, if \$3·75 was charged for the policy and the preliminary survey?

16. For what sum was a house insured if the premium paid was \$17·50 and the rate of insurance $\frac{7}{8}\%$?

17. For what sum was a shop insured if the rate of insurance was 65 cents per \$100 and the premium paid was \$81·25.

18. A fire insurance company received $350 for insuring a factory at $1\frac{1}{4}\%$ premium, and charged $\frac{7}{8}\%$ for insuring a less hazardous property of the same valuation as the factory. What was the amount of the premium paid on the second property?

19. A merchant owns $\frac{2}{3}$ of a steamship and insures $\frac{3}{4}$ of his interest at $\frac{5}{8}\%$, paying $337·50 premium. What was the value of his interest in the steamer? If during the continuance of the policy, the vessel be damaged in a collision to the extent of $35,000, what sum will the merchant be entitled to receive from the insurance company?

20. The invoice price of a shipment of goods is $1845. The shipper wishes to insure the goods for such a sum as will, in case of loss, cover both invoice price and amount of premium. For what sum should the shipment be insured if the rate of insurance is $\frac{3}{8}\%$?

21. The value of a consignment is $4250. For what sum should it be insured that the owner may receive both the value of the consignment and the amount of the premium in case of total loss, the rate of insurance being 55 cents per $100?

22. For what sum should a cargo worth $18,750 be insured to cover the value of the cargo, the cost of insurance at $\frac{7}{8}\%$ and $2·50 for the policy and broker's charges?

23. A cargo of wheat invoiced at $9930 is insured for $10,000 which sum covers not only the invoice value of the wheat but also the premium paid and $5 for expenses. What was the rate per cent. of the insurance?

24. A shipment of goods is insured for $6000, which sum covers the value of the goods, the premium at $1\frac{1}{8}\%$ and $2·50 for expenses. What was the value of the goods?

25. A warehouse with its contents was insured for $75,000, and was damaged by fire and water to the extent of $115,000. Of the risk, $50,000 was taken by the London Fire Insurance Co., $30,000 by the Imperial, $25,000 by the Phœnix, and the remainder in equal parts by the Royal Canadian and the Ontario Mutual. What was the net loss of each company if the rate of premium paid was $1\frac{1}{8}\%$, but the agents were allowed to retain 10% of the premiums?

26. A factory with its contents was insured for $70,000. Of this sum $30,000 was on the building which was worth $45,000; $15,000 was on the machinery which was worth $20,000, and the remaining $25,000 was on the stock which was worth $40,000. A fire occurred by which the building was damaged to the extent of $12,000 and the machinery to the extent of $15,000 and the stock was totally destroyed. How much was the claim against the insurance company if the risk was covered by an ordinary policy? How much was it if the policy contained an "average clause?"

COMMISSION.

An **Agent** is a person authorized to transact business for another. The person for whom the agent transacts business is called his *Principal*.

Commission is the charge made by an agent for transacting business.

The **Gross Proceeds** of a sale or of a collection is the total amount received by an agent for his principal.

The **Net Proceeds** of a sale or of a collection is the sum due the principal from the agent, after deducting his commission and all other charges. These charges include freight, handling, storage, advertising, and such like.

Commission is usually reckoned at a rate per cent. on the gross proceeds of sales and collections, on the prime cost of purchases, and on the net amount of investments.

EXERCISE LXXIII.

1. An agent bought \$750 worth of tea. Find the amount of his commission at 3%. At 1%. At $4\frac{1}{2}$%. At $\frac{1}{2}$%. At $\frac{7}{8}$%.

2. An agent sold a house for \$6750. Find the amount of his commission at 2%. At $1\frac{1}{4}$%. At $\frac{3}{4}$%. At 0·7%.

3. What sum will a principal need to remit to his agent to buy \$4750 worth of flour if the agent's rate of commission be 1%? $2\frac{1}{2}$%? $3\frac{1}{3}$%? $1\frac{5}{8}$%?

4. If an agent collect \$468 on a commission of $2\frac{1}{4}$%, what sum will be due from him to his principal?

5. An agent charged \$29.25 for collecting \$1300. What was his rate of commission?

6. An agent sold a house and lot for \$7500, and remitted \$7340·62 to his principal. What rate of commission did he charge?

7. A commission merchant sold 4000 yd. of white cotton at $7\frac{1}{2}$c. a yd. What sum should he remit his principal, his commission being at the rate of $1\frac{3}{4}$%, and the expenses other than commission amounting to \$45.60?

8. A merchant sent his agent \$5151 to invest for him in dried fruit. What sum should the agent invest, his commission being at the rate of 1%? At the rate of 2%? At the rate of $2\frac{1}{2}$%?

9. A broker is offered a commission of $5\frac{1}{2}\%$ for selling wool and guaranteeing payment, or a commission of $3\frac{3}{4}\%$ without guaranteeing payment. He accepts the $5\frac{1}{2}\%$ and guarantees payment. The sales amount to \$17,000, and the bad debts to \$295.50. How much did he gain by choosing the $5\frac{1}{2}\%$?

10. An agent sold 6 mowing-machines at \$120 each, and 12 at \$140 each. He paid for transportation \$72, and, after deducting his commission, remitted \$2208 to his employer. What was the rate of commission?

11. A man allows his agent 5% of his gross rentals, and receives a net rental of \$3488·40. If the gross rental is 6% of the value of the property, what is the value of the property?

12. On a debt of \$1725 a creditor receives a dividend of 60%, on which he allows his attorney 5%. He receives a further dividend of 25%, on which he allows his attorney 6%. What is the net amount that he receives?

13. An agent sold a quantity of cotton amounting to \$7317·83, and charged a commission of $2\frac{1}{2}\%$. He was instructed to invest the proceeds in dry goods, after deducting a commission of $1\frac{1}{2}\%$ on the amount so expended. What was his total commission?

14. An agent sold 300 bales of cotton, averaging 462 lb. to the bale, at 15·7 ct. per lb., his commission being 25ct. per bale, and the charges being \$161. He purchased for the consignor dry goods amounting to \$2576·37, charging a commission of $1\frac{1}{2}\%$. How much was still due the consignor?

15. A commission merchant sold a consignment of bacon at $11\frac{1}{2}$ ct. per pound and invested the proceeds, less his commission, in tea at 38 ct. per pound. His commission on the two transactions at the rate of 5% on the sale of the bacon and 2% on the purchase of the tea amounted altogether to \$52·50. How many pounds of bacon did he sell and how many pounds of tea did he buy?

16. A miller sends 4000 bbl. of flour to a commission merchant with instructions to sell the flour and remit the net proceeds by draft. The consignee pays \$462·40 for freight and other expenses, sells the flour at \$6·75 per barrel, charges 3% commission and pays $\frac{1}{4}\%$ premium for draft. Find the amount of the draft.

17. The owner of certain property pays $2\frac{1}{2}\%$ for collecting his rents, insurance and repairs cost him $6\frac{3}{4}\%$ of his *net* income but on this sum he pays no tax, his income tax at $17\frac{1}{2}$ mills on the dollar amounts to \$153·73. Find the gross rents from his property.

18. An agent sold a consignment of boots and shoes for \$3825 and invested the proceeds, less his commission, in leather. His total commission amounted to \$150. What rate did he charge, the rates on both sale and purchase being the same?

TRADE DISCOUNT.

Discount *is an abatement or reduction from the nominal price or value of anything;* as, for example, from the catalogue or list price of an article, from the amount of a bill or invoice of goods or of a debt, or from the face value of a promissory note.

The **Rate of Discount** is usually stated as a rate per cent. of *the amount from which the discount is made.*

Thus, a discount of 20% off \$146 means that ·20 of the \$146 is to be deducted from it

·20 of \$146 = \$29·20;
\$146 − \$29·20 = \$116·80.

Trade Discounts are reductions made from the catalogue or list prices of goods.

In some branches of business the manufacturers and the wholesale dealers catalogue their goods at fixed prices, usually the retail selling price, and then allow retail dealers reductions or discounts from these catalogue prices. These discounts generally depend on the amount of the purchase and the terms of payment, whether cash or credit. By varying the rate of discount, the manufacturer can raise or lower the price of his goods without issuing a new catalogue.

Very often two or even more successive trade discounts are to be deducted. In such cases the *first* rate denotes a percentage of *the catalogue price;* the *second* rate denotes a percentage of *the remainder after the first discount has been made;* the *third* rate, a percentage of *the remainder after the second discount has been made;* and so on

Thus, discounts of 20% and 5% in succession off any amount, or, as it is generally expressed in business, *20 and 5 off*, means that ·20 of the amount is to be deducted from it, and then from the remainder ·05 of that remainder is to be taken.

Example.—Find the net cost of goods amounting per catalogue price to \$840, subject to 20 and 5 off.

\$840 = Catalogue price.
·20 of \$840 = 168
\$672 = Proceeds of 1st discount.
·05 of \$672 = 33·60
\$638·40 = Proceeds of 2nd discount = *Net Cost.*

EXERCISE LXXIV.

Find the net cost of goods invoiced at—

1. $440, subject to 15 off.

2. $750, subject to 20 and 15 off.

3. $320, subject to 35 and 15 off.

4. $240, subject to 25 and 7½ off.

5. $360, subject to 25, 10 and 5 off.

6. $144·60, subject to 25, 15 and 12½ off.

7. $435·25, subject to 30, 22½ and 12½ off.

8. The gross amount of a bill of goods was $445.50, and the rates of successive discounts were 25% and 15%. Find the net amount.

9. Find the difference between a single discount of 45% off and successive discounts of 25% and 20% off a bill of $500.

10. What single discount is equivalent to successive discounts of 25%, 11⅑% and 10%?

11. A merchant buys goods amounting per catalogue price to $180.60, subject to 25 and 10 off, and he sells them at catalogue prices. Find the amount and the rate of his profit.

12. A sewing machine agent buys machines at a discount of 25, 10 and 10, and sells them at 10% advance on catalogue prices. Find his rate of profit.

13. Purchased goods amounting to $12,464.40. Sold from them in 92 days $11,631.20. Balance of goods remaining unsold, $5,760.15. Required the total gain, the average daily sales (Sundays excepted), the average daily profits, and the average gain per cent.

14. Sold merchandise at an advance of 30% on cost. My customer failed in business, and I lost 25% from the selling price. What was the net gain or net loss per cent.?

15. A merchant marked his goods at 25% advance on cost, but concluding to give up business he sold his stock at 20% discount from the marked price of the goods. What was his gain or his loss per cent.?

16. An agent receives $14,000 to invest in wheat. How many bushels at 85c. ought he to buy for his principal—1st, if his commission be at the rate of ¾%; 2nd, if it be at the rate of ¾ ct. per bushel? *(In each case work to the* nearest *bushel.)*

17. An agent sold a consignment of sugar, charging 2½% commission. He invested part of the proceeds in 200 bbl. of flour at $6·50 per bl., charging 2% commission; and after deducting $114 for expenses other than his commission, he remitted to his principal the balance, which was $900. For how much did he sell the sugar?

BANK DISCOUNT.

A **Promissory Note** (often called briefly a NOTE) is a written promise to pay, unconditionally, on demand or at a fixed or a determinable future time, a specified sum of money, to a particular person named in the note, or to a person named or his order, or to bearer.

A note which is, or on the face of it purports to be, both made and payable within Canada, is an inland note : any other note is a foreign note.

The **Maker** of a note is the person who signs the promise.

The **Payee** is the person to whom or to whose order the note is made payable.

The **Holder** or Bearer of a note is the person who lawfully possesses it.

The **Face Value** (or simply the FACE) of a note is the sum of money (exclusive of interest) which the maker promises to pay.

A promissory note may be made by two or more makers, and they may be liable thereon jointly, or jointly and severally according to its tenor. If a note runs "I promise to pay," and is signed by two or more persons, it is deemed to be their joint and several note.

An **Indorser** of a note is a person who writes his name on the back of the note. By so doing he guarantees its payment and becomes responsible therefor, unless when indorsing he writes above his signature the words "without recourse." A note payable to *order* must be endorsed by the payee when transferred to anyone else, but a note payable to *bearer* need not be indorsed.

A special indorsement specifies the person, called the *indorsee*, to whom, or to whose order, the note is to be payable.

An indorsement in blank specifies no indorsee, and a note so indorsed becomes payable to bearer. When a note has been indorsed in blank, any holder may convert the blank indorsement into a special indorsement by writing above the indorser's signature a direction to pay the note to or to the order of himself or some other person.

An endorsement is restrictive which prohibits the further negotiation of the note or which expresses that it is a mere authority to deal with the note as thereby directed, and not a transfer of the ownership thereof, as, for example, if a note be indorsed "Pay D only," or "Pay D for the account of X," or "Pay D or order for collection." A restrictive indorsement gives the indorsee the right to receive payment of the note and to sue any party thereto that his indorser could have sued, but gives him no power to transfer his rights as indorsee unless it expressly authorise him to do so. Where a restrictive indorsement authorises further transfer, all subsequent indorsees take the note with the same rights and subject to the same liabilities as the first indorsee under the restrictive indorsement.

A **Negotiable Note** is one which may be sold or transferred by the payee to anyone else ; and a note is negotiated when it is transferred from one person to another in such a manner as to constitute the transferee the holder of the note. A negotiable note may be payable either to order or to bearer. A note is payable to bearer which is expressed to be so payable, or on which the only or last indorsement is an indorsement in blank. A note is payable to order which is expressed to be so payable, or which is expressed to be payable to a particular person, and does not contain words prohibiting transfer or indicating an intention that it should not be transferable. Where a note, either originally or by indorsement, is expressed to be payable to the order of a specified person, and not to him or his order, it is nevertheless payable to him or his order, at his option.

A note payable to bearer is negotiated by delivery. A note payable to order is negotiated by the indorsement of the holder completed by delivery.

Where the holder of a note payable to bearer negotiates it by delivery without indorsing it, he is called a *transferor by delivery*. A transferor by delivery is not liable on the instrument. A transferor by delivery who negotiates a note thereby warrants to his immediate transferee, being a holder for value, that the note is what it purports to be, that he has a right to transfer it,

and that at the time of transfer he is not aware of any fact which renders it valueless.

When a note contains words prohibiting transfer, or indicating an intention that it should not be transferable, it is valid as between the parties thereto, but it is not negotiable.

Where a note is in the body of it made payable at a particular place, it must be presented for payment at that place in order to render the maker liable; in any other case, presentment for payment is not necessary in order to render the maker liable. Presentment for payment is necessary in order to render the indorser of a note liable. Where a note is in the body of it made payable at a particular place, presentment at that place is necessary in order to render an indorser liable; but when a place of payment is indicated by way of memorandum only, presentment at that place is sufficient to render the indorser liable, but a presentment to the maker elsewhere, if sufficient in other respects, will also suffice.

Maturity (properly DATE OF MATURITY) is the day on which the note becomes legally due. Where a note is not payable on demand, the day on which it falls due is determined as follows:

Three days called days of grace, are, in every case where the note itself does not otherwise provide, added to the time of payment as fixed by the note, and the note is due and payable on the last day of grace. Whenever the last day of grace falls on a legal holiday or non-juridical day in the Province where any such note is payable, then the day next following, not being a legal holiday or non-juridical day in such Province, is the last day of grace.

A note is payable on demand, which is expressed to be payable on demand, or on presentation, or in which no time for payment is expressed.

Where a note is payable at a fixed period after date, after sight, or after the happening of a specified event, the time of payment is determined by excluding the day from which the time is to begin to run and by including the day of payment. The term "Month" in a note means the calendar month. Every note which is made payable at a month or months after

date becomes due on the same numbered day of the month in which it is made payable as the day on which it is dated—unless there is no such day in the month in which it is made payable, in which case it becomes due on the last day of that month—with the addition, in all cases, of the days of grace.

A note is not invalid by reason only that it is antedated or post-dated, or that it bears date on a Sunday.

A **Draft** or **Bill of Exchange** is a written order by one person (called the Drawer) directing a second person (called the DRAWEE) to pay, unconditionally, on demand or at a fixed or a determinable future time, a specified sum of money (called the FACE or PAR) to a third person (called the PAYEE) or to the payee's order, or to bearer.

Bills of exchange are negotiable or non-negotiable upon the same conditions and are subject to the same indorsements as promissory notes. The date of maturity of a bill of exchange is ascertained in the same manner as that of a note.

Bank Discount *is a deduction made from the face value of a note or a draft for cashing it or buying it before maturity.*

The **Term of Discount** is the time between the date of the discounting and the date of maturity.

The **Rate of Discount** is the percentage of the FACE VALUE which would be deducted if the term of discount were ONE YEAR. This is the *nominal* rate, the ACTUAL or EFFECTIVE RATE is that fraction of the nominal rate which the term of discount is of one year.

Exchange is a charge made for collection in cases in which the place of payment of the note or the draft is not the place of discount. The rate of exchange is generally from $\frac{1}{8}$ to $\frac{1}{4}$ of 1% of the face value, but if the face value is less than $100, the full exchange on $100 is usually charged.

The **Proceeds** of a note is the sum of money received for it on discounting it. It is equal to the sum due at maturity less the discount and the exchange.

Example.—A note for $712·40, dated July 30th and payable in 3 months from date, was discounted on Aug. 3rd at 7%, exchange $\frac{1}{8}$%. Find the proceeds.

Maturity is 3 mo. 3 da. from July 30th = Nov. 2nd.

Term of discount is from Aug. 3rd to Nov. 2nd = 91 da. = $\frac{91}{365}$ yr.

	$712·40	= Face.
	·07	= Rate.
7% of $712.40 =	49·8680	= Discount for 1 yr.
	$\frac{91}{365}$	= Term of Discount.
$\frac{91}{365}$ of $49·868 =	12·43	= Discount for 91 da.
$\frac{1}{8}$ of ·01 of $712 =	·89	= Exchange.
	$13·32	= Total deduction.
$712·40 − $13·32 =	$699·08	= *Proceeds.*

EXERCISE LXXV.

Find the date of maturity, the term of discount, the bank discount and the proceeds in the following cases :—

Face of Note.	*Date of Note.*	*Time.*	*Date of Discount.*	*Rate of Discount.*
1. $250.	3 June, 1886.	90 da.	5 June.	6%.
2. $470.	25 Ap., 1885.	60 da.	1 June.	7%.
3. $187·50.	14 Sept., 1883.	3 mo.	23 Sept.	5%.
4. $68·75	27 Feb., 1887.	90 da.	4 March.	6%.
5. $983·38.	28 Jan., 1888.	2 mo.	2 Feb	8%.

297\frac{65}{100}$. LONDON, *3 Jan'y, 1887.*

Ninety days after date I promise to pay James Thomson or order Two Hundred and Ninety-seven $\frac{65}{100}$ Dollars at the Mutual Savings Bank here. Value received.

HIRAM JONES.

6. Find the proceeds of the above note discounted in Toronto on 9th Jan'y, 1887, at 7% ; exchange $\frac{1}{4}$%.

714\frac{85}{100}$. SARNIA, *27 Nov., 1886.*

Four months after date we jointly and severally promise to pay to the order of John G. Williams & Co. Seven Hundred and Fourteen $\frac{85}{100}$ Dollars for value received.

HENRY JAMIESON.
THOMAS DORAN.

7. Find the proceeds of the above note discounted at Hamilton on 12 Dec., 1886, at 7% ; exchange 15c. per $100 or fraction thereof.

$\$339\frac{45}{100}$. PEMBROKE, *3 March, 1886.*

At thirty days' sight pay to the order of Brown, Jones & Co., of Kingston, Three Hundred and Thirty-nine $\frac{45}{100}$ Dollars for value received, and charge to the account of

LE MOINE & PETERSON.

To GREER & HENDERSON,
Kingston.

8. Find the proceeds of the above draft discounted at 8%; exchange $\frac{1}{8}$%.

9. Complete the following discount sheet by filling in the blanks; rate of discount 7%, of exchange $\frac{1}{8}$%:—

BANK OF THE YORKTOWN DISTRICT.

TORONTO, *4 May, 1887.*

Bills Discounted for SANDERS, REDFORD & Co.

No.	Drawee.	Where Payable.	When due.	Days to run.	Gross Am't.	Interest.	Ex-c'ge.	Pro-c'ds.
1	Alex. Blatchford..	Stratford..	July 5		$445 60			
2	Fred Meade & Co.	Parkhill...	July 17		149 80			
3	Geo. Hart & Co...	Berlin.....	Aug. 12		265 30			
4	Geo. R. Tighe.....	Guelph....	Aug. 26		514 09			
5	Ab. S. Lewis & Co.	Chatham..	Sept. 3		390 34			

Examined.....

10. Draw up and fill in a discount sheet for the following, arranging the drafts in order of maturing:—Messrs. Jones & Brown, wholesale merchants, Montreal, take the following drafts to their bank on the 17th August, 1887, to be discounted and the net proceeds placed to their credit: One at 30 days from date on Wm. Brown, Brockville, for $260·50; one at 90 days from date on A. B. West, Perth, for $114·40; one at 10 days from date on S. B. Wood & Co., Brantford, $440·25; one at 60 days from date on R. J. Stanford, Ottawa, $54·12: one at 15 days from date on H. C. Bleasdell & Co., London, $95·30; one at 6 days on J. K. Smith & Co., Hamilton, $314·65. How much should the bank place to Jones & Brown's credit, allowing the rate of discount to be 7%? Exchange $\frac{1}{4}$%.

11. A note for $250 was discounted 40 days before maturity and the proceeds were $247·80. What was the rate of discount, there being no exchange?

INTEREST.

Interest is the sum which the lender of money charges the borrower for the use of the sum borrowed, or which a creditor charges a debtor for allowing his debt to remain unpaid after it has become due.

The **Principal** is the sum borrowed or due.

The **Amount** is the sum total of principal and interest.

The **Rate of Interest** is always expressed as the rate per cent. of the principal which would be charged for its use for ONE YEAR.

The **Time** is the period, *expressed in years*, for which interest is reckoned.

Simple Interest is interest reckoned on the original principal and on it alone for the whole term during which that principal bears interest.

Compound Interest is interest which is reckoned for stated periods and added at the end of each period to the principal on which it was reckoned, the amount or sum total of principal and interest at the end of each period becoming the principal for the succeeding period.

It is as if the original principal had been loaned at simple interest for the first period, then the amount from that period loaned for the next period, the second amount loaned for the third period and so on, a new loan of the sum total of principal and all accrued interest being entered upon at the beginning of each period. Thus compound interest reckons interest upon interest.

The names **Annual, Semiannual, Quarterly** and **Monthly Interest** are applied to interest which is payable at the end of each year, half-year, quarter-year or month, as the case may be, throughout the time during which the principal bears interest.

Annual or other periodically payable interest differs from simple interest in that it is to be paid at stated intervals while simple interest is not due and collectible until the principal matures.

Annual or other periodically payable interest differs from compound interest in that it like simple interest is reckoned on the original principal and on it alone while compound interest is computed for each period on the original principal increased by all accrued interest. In effect, however, periodically paid interest is equivalent to compound interest, for the borrower loses the use of the money he pays as interest at the end of each period and the lender gains the use of it, and the value of this use is assumed to be interest at the rate paid on the principal. Hence, *in calculations concerning periodic payments, the methods, not of simple but of compound interest, should be employed.* For example, it is usual with savings banks which pay annual interest to credit each depositor at the end of every interest year with all interest on his deposit accrued but undrawn, treating such interest as a new deposit, the net result being that the banks pay compound interest.

If interest which is by agreement to be paid at specified intervals, is not so paid, and the lender has to collect it by process of law, the courts have authority to grant at their discretion simple interest on the accrued periodic interest. The interest upon interest, if thus granted, is styled *damages* and its maximum rate is the *legal rate* of six per cent. per annum.

The **Present Worth** *at a specified rate of interest* of a bill or a promissory note is the sum of money which put out at interest at the specified rate will when the bill is due or the note matures amount to the sum due on bill or note.

SIMPLE INTEREST.

Problems in simple interest involve the consideration of principal, rate, time, interest and amount; and any three of these being known the other two may be determined, for by definition:

1°. *The simple interest is the continued product of the principal, the rate per* unit *and the numeric of the time.*

2°. *The amount is the sum of the principal and the interest.*

Example.—Find the amount of $756·80 in four months from 23rd May at 7%.

From 23rd May to 23rd Sept. = 123 dy. = $\frac{123}{365}$ yr.

$$\begin{array}{rl} \$756{\cdot}80 & = \text{Principal.} \\ 7\% = \quad {\cdot}07 & = \text{Rate of interest per year.} \\ \hline 52{\cdot}9760 & = \text{Interest for 1 year.} \\ \frac{123}{365} & \\ \hline 17{\cdot}85 & = \text{Interest for 123 days.} \\ 756{\cdot}80 & = \text{Principal.} \\ \hline \$774{\cdot}65 & = \text{Amount.} \end{array}$$

$\frac{123}{365}$ of $52·976 =

EXERCISE LXXVI.

Find the simple interest on and the amount of :—

1. $150 for 2 yr. at 6%.
2. $215 for 3 yr. at 5%.
3. $347·50 for 4 yr. at 4%.
4. $167·80 for 1½ yr. at 6%.
5. $84·75 for 3¼ yr. at 4½%.
6. $188·65 for 146 da. at 7%.
7. $375 for 93 da. at 6%.
8. $176·40 for 126 da. at 5½%.

9. An account for $446·50 due on the 21st of May was not paid till 13th August. Find the interest on it for that period at 6%.

10. A merchant purchased on the 17th September, 1887, goods amounting to $796.40. He was allowed 3 months' credit on the purchase, after which he was charged interest at the rate of 7½% per annum. He did not pay his account till the 3rd March, 1888. Find its amount at that date.

11. A merchant purchased on the 13th February, 1885, goods amounting per catalogue prices to $863·45, subject to 25 and 15 off. He is allowed one month's credit from date of purchase, after which he is charged interest at 9%. Find the amount of the account on 19th July, 1885.

12. At what rate would $125 yield $15 interest in 2 yr.?

13. At what rate of interest would $225 amount to $231 in 139 da.?

14. At what rate will a sum of money at simple interest double itself in 20 years?

15. At what rate will a sum of money at simple interest quadruple itself in 50 years?

16. In what time will $273·85 yield $28·86 simple interest at 6%?

17. In what time would $401·50 amount to $410.30 at 6¼%?

18. A debt of $175 became due on 13th June after which date interest was charged at the rate of 7%. When the debt was paid the interest accrued on it was $4·10. When was the debt paid?

19. In what time will a sum of money double itself at 5% simple interest?

20. In what time will a sum of money triple itself at 8% simple interest?

21. The proceeds of a note for $137·50 discounted 40 days before maturity, were $136·30. What was the rate of discount charged on the face of the note and what was the rate of interest paid on the proceeds?

22. Find the discount off $385·77 due 86 days hence, (i) at 8% *discount*, (ii) at 8% *interest*. Show that the difference between amounts (i) and (ii) is the interest at 8% on (i) or the discount at 8% off (ii).

23. What rate of interest is equivalent to 10% discount, the term of discount being one year?

24. What rate of interest is equivalent to 10% discount, the term of discount being 95 days?

COMPOUND INTEREST

Compound Interest is interest which is computed for stated periods and added at the end of each period to the principal on which it was computed, the sum-total of principal and accrued interest at the end of each period becoming a new principal on which interest is computed for the next succeeding period.

The interest is said to be compounded annually, semi-annually, quarterly, monthly,.....according as the addition of interest to principal is made every year, half-year, quarter-year, month, or other interval.

In stating the rate of interest, one year is taken as the unit of time but it is not expressed, and the rate is reduced to an annual rate as if it were for simple interest. Thus 4% compounded semi-annually does not mean 4% per half-year but 2% per six months, the full phrase being,—"4% per annum but compounded semi-annually." A rate expressed in this way as if it were a simple interest rate is called a **nominal rate** to distinguish it from the actual or effective rate. A nominal rate of 6% compounded quarterly is an actual rate of $1\frac{1}{2}$% per quarter-year, and a nominal rate of 12% compounded monthly is an actual rate of 1% per month.

Example.—If $1250 deposited in a savings-bank, draw interest at 4% payable semi-annually, the interest accrued and due at the end of the first half-year will be ·02 of $1250 which is $25·00. If this $25·00 be not drawn it will be placed to the credit of the depositor, making his deposit $1275.

The interest for the second half-year will be computed on the increased deposit and will therefore be ·02 of $1275 which is $25·50. If this $25·50 be not drawn it will be placed to the credit of he depositor, making his deposit $1300·50 at the beginning of the third period of six months.

The interest for the third half-year will be computed on the $1300·50 deposit and will therefore be ·02 of $1300·50 which is $26·01. This sum, if it be not drawn, will be added to the $1300·50 making a total of $1326·51 at the credit of the depositor at the end of 18 months.

Thus $1250 at 4% interest compounded semi-annually will in a year and a half amount to $1326·51; and the compound interest at the specified rate and for the stated time will be $1326·51 − $1250 = $76·51.

Computation.

$1250 = original amount or principal.
1·02 = *rate* of increase in amount.

25·00
1250

$1275 = amount at end of 1st period.
1·02

25·50
1275

$1300·50 = amount at end of 2nd period.
1·02

26·0100
1300·50

$1326·51 = amount at end of 3rd period.

EXERCISE LXXVII.

Find the amount and the compound interest of :—

1. $800 for 3 years at 5% compounded annually.
2. $425 for 4 years at 4% compounded annually.
3. $250 for 2 years at 6% compounded semi-annually.
4. $366·67 for 2½ years at 4% compounded semi-annually.
5. $722·50 for 1½ years at 4% compounded quarterly.

Find correct to six significant figures the amount of $1 at compound interest at 6% for one year, interest compounded:

6. annually. 7. semi-annually. 8. quarterly.

Find correct to six significant figures the rate of increase in the amount of $1 at 5% interest compounded annually for

9. three years. 10. five years. 11. seven years.

Find correct to six significant figures the rate of increase in the amount of $1 at 4% interest compounded quarterly for

12. one year. 13. two years. 14. three years.

PARTIAL PAYMENTS.

A **Partial Payment** is a payment of only a part of a debt and its accrued interest.

A **Receipt Indorsement** is an acknowledgment of the receipt of a partial payment written on the back of a note, mortgage or other documentary evidence of debt, stating the amount and the date of the payment.

When partial payments have been made on an interest-bearing note or other obligation, the balance unpaid and due at any given date may be found as follows:—

Find the interest on the principal from the date of the note or other obligation to the date of the first partial payment.

(a) If the first partial payment is equal to or exceeds the interest thus found, subtract the first payment from the sum of the principal and its accrued interest, and consider the remainder as a new principal.

(b) If the first partial payment is less than the interest thus found, find the interest on the principal to the date of the next or of the earliest subsequent partial payment at which the sum of the payments equals or exceeds the interest due at such date, and

subtract the sum of the payments to that date from the sum of the principal and its accrued interest to that date, and consider the remainder as a new principal.

Similarly find the interest on the new principal to the date of the next partial payment. If that payment be equal to the interest thus found or if it be greater than the interest, proceed as in (a); but, if the payment be less than the interest, proceed as in (b). So continue to the date of settlement.

A partial payment in excess of the accrued interest will have the effect of reducing the principal, since, after discharging such accrued interest, there will remain a surplus to be so applied.

A partial payment less than the accrued interest will not reduce the principal since such payment is not sufficient to discharge the accrued interest *which must first be paid.* No new principal should exceed the preceding principal, for such excess could arise only by the addition of interest to that preceding principal, and the effect would be to compute interest on interest, in computing the interest on the new principal.

EXERCISE LXXVIII.

1. On a note for $620 on demand, dated Oct. 18, 1888, and drawing 6% interest are indorsed the following payments: Nov. 26, 1888, $47·50; Dec. 28, 1888, $108·93; Feb. 11, 1889, $216·18; June 6, 1889, $60·10; Sep. 2, 1889, $183·25. How much was due on the note on Nov. 11, 1889?

2. On a mortgage for $3750 dated May 16, 1887, and bearing interest at 6%, there were paid May 16, 1888, $350; Sept. 18, 1888, $280; Jan. 22, 1889, $750; May 16, 1889, $925; Oct. 31, 1889, $500. What sum was due on the mortgage on Jan. 2, 1890?

3. How much was due on the following note, on Oct. 31, 1889?

$850. Toronto, Oct. 31, 1887.

For value received, I promise to pay Alex. Thompson or order, on demand, Eight hundred and fifty Dollars, with interest from date at six per centum. John Stuart.

On this note the following payments were indorsed.

April 20, 1888, $125. Jan. 21, 1889, $75.
Nov. 20, 1888, $125. July 20, 1889, $425.

AVERAGING ACCOUNTS.

When one person owes another several amounts due at different times, the date on which all these debts may be discharged by payment of their sum, without loss of interest to either the debtor or the creditor is called the AVERAGE DATE or **Equated Time.**

Example.—A bought goods of B as follows:—May 17, $200 at 30 days' credit; June 3, $250 at 60 days' credit; June 12, $210 at 90 days' credit. On July 5, A paid B $300 on account. Find the equated time for paying the balance.

Had A paid B $200+$250+$210=$660 on May 17, B would have gained the interest on $200 for 30 days, the interest on $250 for 77 days and the interest on $210 for 116 days.

But if A delay from May 17 to July 5 to pay $300 of the $660, B's gains will be reduced by the interest on the $300 for 49 days, the number of days of delay.

And if A defer the payment of the $360, balance of the $660, until the equated date, B will lose the balance of the interest he would have gained had all the payments been made on May 17.

Interest on $200 for 30 da. = Int. on $1 for 30 da. × 200 = Int. on $1 for 6000 da.
" 250 " 77 " = " " 77 " × 250 = " " 19250 "
" 210 " 116 " = " " 116 " × 210 = " " 24360 "
" $660 " " 49610 da.
" 300 for 49 da. = Int. on $1 for 49 da. × 300 = " " 14700 "
" $360 for equated time = " " 34910 da.
" $1 for 34910 da. = Int. on $360 for $\frac{34910}{360}$ da. =
Int. on $360 for 97 da.

Therefore the *equated time* = 97 days after May 17 = Aug. 22.

If any of the items include cents omit the cents in the calculation, and take the nearest number of dollars to the amounts of the items.

The method of determining the equated time of an account, which is exhibited in the preceding solution, is based on the assumption that what the debtor gains by retaining certain sums after they become due he loses by paying other sums before these become due, but as both gains and losses are computed on the full amounts of the items, while the actual gain is the interest

on the amounts of the deferred payments and the actual loss is the interest on the present worth of the anticipated payments, it is evident that the solution is not absolutely correct. However, in ordinary business transactions, the error is too small to materially affect the result.

EXERCISE LXXIX.

Find the equated date of payment of

1. Sep. 3 $350 @ 60 da.
" $520 @ 90 da.
" $175 @ 30 da.

2. Aug. 27, $325 @ 60 da.
Sept. 20, $280 @ 30 da.
Oct. 31, $784 @ 90 da.

3. On May 2, goods amounting to $1250 were purchased on the following terms; $400 payable in 30 days, $500 payable in 60 days and the balance payable in 90 days. Find the equated date for the payment of the whole bill.

4. A house was sold for $9500 on the following terms, one-third in cash, one-half in twelve months and the remaining sixth in eighteen months. Find the equated time for the payment of the entire sum.

5. On Sep. 19, a commission merchant received a consignment of 600 barrels of apples. He sold 120 barrels at $2·25 on Sept. 24; 75 barrels at $2·30 on Sep. 27; 150 barrels at $2·40 on Oct. 7th; 150 barrels at $2·35 on Oct. 22; and the balance at $2·20 on Nov. 18. Find the equated date of the total sales.

6. Henry Simpson sold A. Thomson & Co. merchandise as follows: Sep. 1, 225 bbl. flour @ $6, on 30 days' credit; Sep. 9, 180 bbl. of pork averaging 208 lb. @ 11¼ ct., on 60 days' credit; Sep. 17. 150 doz. eggs @ 16 ct. per dozen on 2 months' credit; Oct. 7, 572 lb. bacon @ 13½ ct. on 3 months' credit; Nov. 10, 460 lb. butter @ 21½ ct. on 90 days credit. Find the equated date for the payment of the sum-total of the several bills.

7. A sold to B on March 4, goods amounting to $728 on 3 months' credit. On Ap. 2, B paid $450 on this purchase. Find the equated date when the balance is due.

8. A holds three promissory notes made by B, one is for $245.60 payable in 3 months from Feb. 13, 1889; another is for $425 payable 60 days after date of Mar. 5, 1889; and the third is for $186·25 and is dated Ap. 3, 1889, and payable 90 days after date. On Ap. 17, 1889, B offers to pay $500 on the notes, and give in exchange for them a single note for the balance on them unpaid. When should the single note be payable?

STOCKS AND BONDS.

A **Corporation** or **Incorporated Company** is an association of persons authorized by law to transact business as a single individual. The powers, rights, duties and obligations of a corporation, as such, are distinct from those of the members forming it.

The capital of a corporation or of a public company is usually divided into a definite number of equal parts called **Shares**. A share commonly represents \$100 (or £100) of the original capital of the corporation, but in some cases it represents as low as \$1 (or £1) of it and in other cases as high as \$1000 (or £1000) of it.

Any number of shares in a corporation or any amount of its capital is called **Stock**, but in the United States this term is also used distinctively for shares of \$100 each, shares of \$50 and \$25 being called *half-stock* and *quarter-stock* respectively.

The proprietors of shares in a corporation or in a public company are called **shareholders** or **stockholders**. Each owner of stock may sell his shares or otherwise transfer them to another person without the consent of the other shareholders.

A **Stock Certificate** is an instrument issued by a corporation, certifying that the holder thereof owns a stated number of shares of the capital stock of the corporation.

The Par Value of a share is the value which is specified upon the face of the certificate for the share, and represents the amount of capital stock for which it was originally issued.

The **Market Value** of a share is the sum for which it can be sold.

Stock is said to be *above par* or *at a premium* when the market value of the shares is greater than their par value; it is said to be *below par* or *at a discount* when the market value of the shares is less than their par value.

A **Dividend** is the part of the net earnings or profits of a corporation or a public company, which is divided among the stockholders thereof. Dividends are usually declared annually,

semi-annually, or quarterly at a specified rate per cent. of the par value of the stock.

Preferred Stock is that part of the capital stock of a corporation on which a specified percentage is payable annually out of the net earnings, before any dividend can be declared on the ordinary stock.

A **Bond** or **Debenture** is a written obligation to pay the holder thereof a certain sum of money at the expiry of a certain term of years, and interest thereon at a specified rate per cent. at stated intervals. Bonds and debentures are issued for money borrowed by the General and Local Governments and by municipal and business corporations. Debentures frequently charge certain specified property with the repayment of the money borrowed on them; in such cases the debentures are practically mortgages on the property.

An **Interest Coupon** is an interest certificate payable to bearer, printed at the bottom of bonds and debentures given for a term of years. There are as many coupons attached to each bond as there are instalments of interest to be paid on it, a coupon for each instalment. Each coupon is cut off and presented for payment when the interest for the period mentioned in it becomes due.

Consols, *i.e.*, Consolidated Annuities are British Government securities bearing 3% interest. These with the other British Government securities for which permanent provision has been made, the most important of which are the Reduced Annuities and New three per cent. Annuities, are in England termed the *Public Funds.*

Rentes (*i.e. Annuities*) are French Government securities bearing various rates of interest.

Stock Brokers are persons who deal in stocks, bonds and similar securities. When a stock broker buys or sells for a principal he charges a commission, technically termed *brokerage*, which ranges, according to circumstances and previous agreement, from $\frac{1}{16}$ of 1% to $\frac{1}{2}$ of 1% of the par value of the securities bought or sold, the most common rate being $\frac{1}{8}$ of 1%. Occasionally special rates are agreed upon and paid.

In England, stock brokers do not deal directly with each other but sell to or buy from stock-jobbers who act for themselves and make their profits out of the turn of the market.

In Canada and the United States, stock quotations usually state only the rates per cent. which the market values of the stocks and bonds quoted bear to their par values; but in England quotations of other than government securities generally give the price per share or per bond.

The following is an illustrative example of a stock report and quotations :—

The closing prices on the Stock Exchange to-day (Aug. 10, 1893), were as follows :

Stocks.	Par Value of Shares	Last ½ year dividend	Toronto		Montreal	
			Sellers per 100	Buyers per 100	Sellers per 100	Buyers per 100
Banks.	$	%				
British North Am. ..	243	3¾	153	....	....	....
Montreal..	200	5	222	214	220	213
Ontario.....	100	3½	114	....	114	....
Molsons..	50	4	165	....	105	....
Toronto..	200	5	245	237½	....	230
Merchants'.....	100	3½	155	149	150½	149
Commerce.....	50	3½	135	133	135	131½
Imperial.....	100	4	180	173	....	....
Dominion.....	50	5	262	....	....	....
Standard	50	4	163	....	....	....
Hamilton.....	100	4	160	152	....	....

It will be seen from this report that in Toronto on Aug. 10, 1893, sellers of Bank of Montreal stock were offering it at the rate of $222 *cash* per $100 *stock* and as each share represents $200 of stock, sellers were really asking $444 per share. The report also shows that buyers of Bank of Montreal stock were offering for it 214% of its par value, *i.e.*, were offering $428 per share for it.

Other forms of report may be seen in the "Financial Columns" of the Toronto and Montreal daily newspapers and in the "Share Lists" of any Stock Exchange.

Example 1.—Find the price at $140\frac{3}{4}$ of 40 shares ($40 each) of Western Assurance Co. stock, brokerage $\frac{1}{8}\%$.

Par value of stock $= \$40 \times 40 = \$1600$.

Rate paid $= 140\frac{3}{4}\% + \frac{1}{8}\% = 1{\cdot}40\frac{7}{8}$.

Cost of stock $= \$1600 \times 1{\cdot}40\frac{7}{8} = \$2254$.

Example 2.—I sold 500 shares of Bank of Montreal stock at $224\frac{3}{4}$ and invested the proceeds in Bank of Commerce stock at $124\frac{1}{4}$, paying $\frac{1}{8}\%$ brokerage on each transaction. Find the increase in my annual income, the Bank of Montreal paying a half-yearly dividend of 5%, the Bank of Commerce a half-yearly dividend of $3\frac{1}{2}\%$.

Par value of B. of M. stock $= \$200 \times 500 = \$100{,}000$.

Rate received $= 224\frac{3}{4}\% - \frac{1}{8}\% = 2{\cdot}24\frac{5}{8}$.

Amount to be invested $= \$100{,}000 \times 2{\cdot}24\frac{5}{8} = \$224{,}625$.

Rate paid for B. of C. stock $= 124\frac{1}{4}\% + \frac{1}{8}\% - 1{\cdot}24\frac{3}{8}$.

Price of 1 share of B. of C. stock $= \$50 \times 1{\cdot}24\frac{3}{8} = \$62{\cdot}1875$.

Number of shares bought is the integral part of

$$\$224625 \div \$62{\cdot}1875$$

which is 3612

and there is $2·75 of cash over.

Par value of 3612 shares of B. of C. stock $= \$180{,}600$.

2 dividends at 5% each on $100,000 of B. of M. stock

$$= \$100000 \times {\cdot}10 = \$10000.$$

2 dividends at $3\frac{1}{2}\%$ each on $180600 of B. of C. stock

$$= \$180600 \times {\cdot}07 = \$12642.$$

Increase of annual income $= \$12642 - \$10000 = \$2642$.

EXERCISE LXXX.

Find the cash value of

1. 25 shares Ontario Bank at 131.
2. 18 " Standard Bank at 137½.
3. 75 " Bank of Toronto at 218.
4. 250 " ($50) Dominion Telegraph Co. at 83¾.
5. 950 " ($100) Canadian Pacific R. R. at 72⅝.
6. 350 " ($24·33⅓) North West Land Co. at 79¾.

7. Sold through a broker 1500 shares ($100) of Jersey Central R. R. stock at 121½, brokerage ⅛%. What were the net proceeds of the sale?

8. Bought through a broker 1600 shares ($100) St. Paul R. R. stock at 69¼, brokerage ⅛%. What was the gross cost of the stock?

9. A speculator bought 36500 shares ($100) Reading R. R. stock at 39¾ and sold them at 40⅜. What was his gain on the transaction?

10. A man bought through a broker 1900 shares ($100) Canada Southern R. R. stock at 54¾ and sold them at 55⅜. What was his net profit on the transaction, brokerage each way ⅛%?

11. A man bought through a broker 7600 shares ($100) of Lake Shore R. R. stock at 107⅝ and sold 2400 shares at 107¾ and the remainder at 107⅛. What was the amount of his losses on the transactions, brokerage being ⅛% each way?

12. A bank declared a dividend of 3½%. How much should a stockholder owning 120 shares ($50) receive?

13. An insurance company declared a dividend of 6%. What rate is that on the market value of the shares which are at 185?

14. Compare the rates on the cash values of 6% on stock at 216 and 3½% on stock at 125.

15. Sold 37 shares ($25) B. and L. Association stock, receiving therefor $1019·81. At what rate was the stock sold?

16. Bought through a broker 750 shares ($50) in the Farmers' Loan and Saving Society paying therefor $43968·75. At what quotation were they bought, brokerage ⅛%?

17. Sold through a broker 215 shares ($50) in the Dominion Savings and Loan Society receiving from him for them $9728·75. At what quotation did the broker sell them, brokerage ⅛%?

18. Bought stock at 197⅝ and sold it at 194⅞, having meanwhile received a dividend of 6% on it. My net gain by the transaction after paying ¼% brokerage each way, is $336. How many shares ($40) did I buy?

19. A man received $495 as dividend at 4½% on his bank stock. He sold 40 shares ($100) at 143¼ and the remainder at 144½, paying ¼% brokerage. What were the net proceeds of the sale?

20. A capitalist had $20,000 to invest. He purchased $8700, par value, of Canadian 4% bonds at 103 and $7300, par value, of Canadian 3½% bonds at 93½ and invested the balance as far as he could in bank stock (shares $100) at 149¼, paying half-yearly dividends of 4% each. What was the gross amount of his investment he paying ⅛% brokerage for buying each class of securities? What was his annual income from these investments? What average rate per cent. per annum did he receive on these investments?

21. The difference between the annual income derived from a certain sum invested in 7% stock at 150 and that from an equal sum invested in 9% stock at 202½, is $40. What is the amount invested in the 7% stock and what is the annual income therefrom?

22. A shareholder receives a dividend of 6% on his stock and pays thereon an income-tax of 16⅔ mills on the dollar. Next year he receives a dividend of 6½% and pays an income-tax of 12½ mills on the dollar. He finds that his income is $830 more in the latter year than it was in the former. How much stock does he hold?

23. A man invests a certain sum in 3% stock at 90 and an equal sum in 4% at 95. Each stock rises 5% in price; the investor then sells out and invests the proceeds of each stock in the other. The stocks fall to their former value and he again sells out at a loss of $3514·75 on the amount realized on his former sales. Find the sum he originally invested.

24. On 13th Sep. a broker bought for a customer 500 shares ($100) of railroad stock at 78¾, and the customer deposited with the broker $5000 as a "margin" for the latter's security against loss by a fall in the price of the stock. On 7th Oct. the stock was sold at 75½. How much of the margin remained after the sale if the broker charged ⅛% brokerage on each transaction and interest at 6% for the exact number of days on the money used in excess of the deposit?

25. If money be worth 5%, what should be the price of 6% bonds which are to be paid off at par 3 years after the date of purchase, the interest on the bonds being payable half-yearly.

CHAPTER IX.

PARTNERSHIP.

A **Partnership** is a voluntary association of two or more persons who combine their money, goods or other property, their labor or their skill, any or all of these, for the transaction of business or the joint prosecution of any occupation or calling, such as the carrying on of any manufacture or trade or the practice of any profession, upon an agreement that all gains and losses shall be shared in certain specified proportions among the persons constituting the partnership.

Such an association is styled a *Firm*, a *Company*, or a *House*, and the persons uniting to constitute the association are called the *Partners of the Firm.*

The **Investment** of a partner in a firm is the money or property contributed by him to the firm.

The **Capital** of a firm is the total of the investments of the partners.

The **Net Gain** within a certain period is the excess of the total gains of a firm over its total losses within the period.

The **Net Loss** within a certain period is the excess of the total losses of a firm over its total gains within the period.

A **Dividend** is the share of the net gain or of any sum divided among the members of a firm or a company, which belongs to any partner. The dividends to the several partners are generally in proportion to their investments.

In a partnership in which the gains and losses are to be divided among the partners in proportion to their investments, to find each partner's share of any net gain or net loss :—

i. *If the investments are contributed for equal times, divide the net gain or the net loss in proportion to the investments.*

ii. *If the investments are contribed for unequal times, multiply each investment by the measure of the length of time during which it was invested and divide the net gain or the net loss in proportion to the products.*

EXERCISE LXXXI.

1. R. Stuart and G. Armstrong enter into partnership and agree to share all gains and losses in proportion to their investments. Stuart contributes $4500 to the partnership and Armstrong contributes $7500. Their net gain at the end of the year is $1750. How much of this sum should each partner receive?

2. Three partners invest respectively $7800, $5750 and $9450 in business, At the end of the first year they find their net gain to be $3156. What is the amount of each partner's share of this gain?

3. Two contractors, G. Rose and W. Crerar, undertake to build a bridge for the sum of $31,500. Crerar supplies the material at a cost of $11,727 and Rose pays the wages of the mechanics and laborers and all other expenses connected with the contract, amounting altogether to $15,645·80. If the profit on the contract is to be divided in proportion to investment, how much of the $31,500 should each partner receive?

4. A. Jones and D. Smith enter into partnership, the former investing $13,500 and the latter investing $22,800, and they agree that Jones shall receive a salary of $2000 a year for managing the business, and that all gains over and above this sum and all losses shall be shared in proportion to their respective investments. At the end of a year their resources are $74,850 and their liabilities are $17,943·86. Find the amount of the interest of each partner at the end of the year.

5. Th. Sinclair, C. Harvey and H. Stevens enter into partnership, Sinclair investing $37,500, Harvey $28,600, and Stevens $24,000, and they agree to share all gains and all losses in proportion to their investments. At the end of the year the resources of the firm are $124,368·50 and the liabilities are $37,429·50. Stevens now wishes to withdraw from the firm and sells to his partners his interest in the business in shares proportional to their interests in it. How much should he receive from each?

6. T. Allan and E. Jamieson engage in business with a joint capital of $19,200 and agree to share gains and losses in proportion to their investments. At the end of a year Allan receives a dividend of $1100 and Jamieson a dividend of $1300. What was the amount of the investment of each?

7. A, B and C enter into partnership and agree to share gains and losses in proportion to their investments. A invests $6000 and B invests $8000. They gain $1800 of which C's share is $792. Required C's investment, and A's and B's gains.

8. D. Rowan, F. Galbraith and J. Munro enter into partnership and agree to share all gains and all losses in proportion to their several investments. They gain $7,500 of which Rowan receives $2100, Galbraith $3100, and Munro the balance. How much did Rowan and Galbraith respectively invest if the amount of Munro's investment was $18,000?

9. Three merchants enter into partnership, the first invests $1855 for 7 months, the second invests $887·50 for 10 months and the third invests $770 for 11 months; and they gain $434. What should be each partner's share of the gain?

10. L, M and N entered into partnership and invested respectively $19,200, $22,500 and $28,300. At the end of 5 months L invested $3800 additional; M, $2500; and N, $3700. At the end of a year the net gain of the firm was found to be $7850. What was each partner's share of this, if all gains and all losses were shared among the partners in proportion to their average investments?

11. Graves and Barr form a partnership, Graves investing $7000 and Barr $8000. At the end of 3 months Graves increases his investment to $9000 but at the end of 5 months more he withdraws $4000 from the business. Barr, 4 months after the formation of the partnership, withdraws $2000 of his investment but 5 months later increases it by $4000. At the end of the year the resources of the firm are $27,850 and its liabilities are $8460. What is the amount of each partner's interest in the business now, the net gain being divided between the partners in proportion to their average investments?

12. Stuart and Moss enter into partnership, Stuart contributing $5000 more capital than Moss. At the end of 5 months Stuart withdraws $2500 of his capital and 2 months later Moss increases his investment by $2500. At the end of their first year of partnership, their assets exceed their liabilities by $24,800 and on dividing their net gain in the ratio of their average investments, Stuart's interest in the business is found to exceed that of Moss by $461·54. Find the amount of the original investment of each.

13. Three companies, A, B and C are to be consolidated on the basis of the relative market values of their stocks. A's capital is $1,000,000 at 90; B's is $750,000 at 70, and C's is $625,000 at 80. The capital of the new company is to be $2,000,000 in shares of $100 each. What amount of this capital should be allotted to each of the old companies and to how much stock in the new company should the holder of 100 shares of the stock of each of the old companies be entitled?

CHAPTER X.

I. APPROXIMATE CALCULATIONS.

In calculations in which fractions occur the greater part of the labor of computation arises in general from the variety of denominators of the fractions. For example, if several fractions of various denominations are to be added together, they must all be reduced to a common denominator, and if one fraction is to be divided by another of different denomination, the terms of the quotient will frequently be much larger than the terms of the dividend. In such cases the labor of computation may often be lessened and the calculations may always be simplified by replacing the fractions by approximately equal decimal numbers, but if we do this we deliberately introduce errors into the calculations. These errors can, however, be kept so small that the result of the computation will be sufficiently near the truth for all practical purposes, for in calculations concerning quantities determined by measurements, it should be remembered that these measurements cannot be made with absolute accuracy. In the measurements of every-day life we are satisfied if we do not err by more than one part in a thousand; in the most careful scientific work it is rarely possible to reduce the error below one part in a million. The results of calculations based on such measurements are necessarily affected by the errors of measurement, and it is therefore a mere waste of time and labor to carry any calculation beyond the degree of accuracy with which measurements can be made. To do so is moreover misleading, for the results then present an appearance of exactness where exactness does not and cannot exist.

The first significant figure in any number is the first digit,—the first figure other than zero,—on the left of the number. For example, in 1·094 yd., 1 is the first significant figure, and in 0·03527 oz., 3 is the first significant figure.

A number is said to be correct to two, three, four..... significant figures if it does not differ from the number that would express the exact value by more than 5 in the second, third, fourth,.....place on the right of the first significant figure.

Example.—If it be said that the length of a certain rod is 39 in. correct to *two* significant figures, it is meant that the actual length of the rod is less than 39·5 in. but is not less than 38·5 in.

If the length be given as 39·4 in. correct to *three* significant figures, it is meant that the actual length is less than 39·45 in. but is not less than 39·35 in.

If the length be given as 39·37 in. correct to *four* significant figures, the actual length may be any from 39·365 in. inclusive to 39·375 exclusive.

A zero may be a significant figure.

Example 1.—The length of a millimetre is 0·001094 yd. correct to four significant figures. Here the zero between 1 and 9 is a significant figure but the three zeros on the left of 1 are not significant figures, they merely indicate that 1, the first significant figure, is the thousandth part of a yard.

Example 2.—The length of the polar diameter of the earth is 41,710,000 ft. correct to five significant figures. Here the zero in the thousands' place is significant but the three zeros on its right are solely locative ; they do not denote the absence of digits in the number expressing the *exact* length of the diameter but merely that these digits have not been determined by the measurements, or at least that they are not here assigned.

The degree of any approximation is measured by the fraction which the error is of the exact value, *i.e.*, by the quotient of the difference between the exact and the approximate value divided by the exact value. *The degree of approximation is* therefore *independent of the unit of measurement.*

Example.—A French metre is equal to 39·3704 in. correct to six significant figures. The length of the metre is therefore at least 39·37035 in. and at most 39·37045 in., consequently the greatest error that can be committed by taking 39·3704 in. for

the exact length of a metre, is 0·00005 in., and the degree of approximation in such case is *at the very worst* $\frac{0{\cdot}00005}{39{\cdot}37035}$ which is less than $\frac{1}{750{,}000}$.

Hence in reducing English measures of length to French measures, or vice versa, at the rate of 39·3704 in. to the metre, the error introduced will be less than 1 part in 750,000.

Different degrees of approximation may be roughly compared by comparing together the number of significant figures known to be correct in each case. Thus if the first three significant figures are known to be correct, the approximation is about ten times as close as it would be if only the first two were known to be correct. "Correct to six significant figures" means an approximation about 1000 times as close as that expressed by "correct to three significant figures."

In expressing mixed numbers and fractions by approximately equal decimal numbers, it will in general be sufficient if the calculations be correct to four or at most to seven significant figures. *Beyond seven figures it is very seldom necessary to go.* Also if one approximate number is to be multiplied by another or to be divided by another, *the result need not be calculated to a greater number of significant figures than are correct in the given numbers.* Hence in all computations with approximate numbers, suitable methods of contracting the operations of multiplication and division should be adopted. The two following examples show how such contractions may be made.

Example 1.—Multiply 453·593 by 91·4392, correct to six significant figures.

Uncontracted Form.		Contracted Form.	
453·593		453·593	
91·4392		91·4392	
40823·37		40823·37	
453 59	3	453 59	..... (*a*)
181 43	72	181 44	..... (*b*)
13 60	779	13 61	..... (*c*)
4 08	2337	4 08	..... (*d*)
9	07186	9	..... (*e*)
41476·18	10456	41476·2	

Write the multiplier under the multiplicand so that the ones' figure of the multiplier may be under the right-hand figure of the multiplicand.

Begin the multiplication by multiplying by 9, the first significant figure of the multiplier. The product will contain seven significant figures; this is one more than the number required to be correct in the complete product, but retain all seven in order to determine the "carriage" to the sixth significant figure when the partial products are added together. Contract the other partial products thus;—

(a) Strike out 3, the right hand figure of the multiplicand, and multiply the resulting 'contracted' multiplicand by 1, the second significant figure of the multiplier.

(b) Strike out 9, the right hand figure of the multiplicand as contracted in *(a)*, and multiply by 4, the third significant figure of the multiplier, carrying 4 from the 9×4 struck out.

(c) Strike out 5, the right hand figure of the multiplicand as contracted in *(b)*, and multiply by 3, the fourth significant figure of the multiplier, carrying 2 from the 5×3 struck out.

(d) Strike out 3, the right hand figure of the multiplicand as contracted in *(c)*, and multiply by 9, the fifth significant figure of the multiplier, carrying 3 from the 3×9 struck out.

(e) Strike out 5, the right hand figure of the multiplicand as contracted in *(d)*, and multiply by 2, the sixth significant figure of the multiplier, carrying 1 from the 5×2 struck out.

The sum of the right hand figures of the partial products is 38. This would give 8 as the seventh significant figure of the total product, but as that product is to be correct to only six significant figures, change 38 to the nearest multiple of 10 which in this case is 40. Now complete the addition of the partial products as in the ordinary uncontracted form.

[Some authors recommend that the figures of the multiplier be written in reverse order, each figure to be placed immediately beneath the right hand figure of its "contracted" multiplicand. The preceding example would in such case be arranged thus:—

453·593
293 419

This is known in England as Oughtred's Method, it having been first

described by William Oughtred in his *Arithmeticæ in Numero et Speciebus Institutio, 1631.* Although frequently rediscovered and described, it has never been generally adopted outside the school-room, being found no more convenient than the method described above, but on the other hand very liable to lead to error.]

Example 2.—Divide 41476·2 by 453·593 and obtain the quotient correct to six significant figures.

Uncontracted Form.		Contracted Form.	
91·4392		91·4392	
453593)41476200·0000		453593)41476209·	
4082337		4082337	
65283|0		65283	
45359|3		45359	.... (*a*)
19923|7·0		19924	
18143|7·2		18144	.... (*b*)
1779|980		1780	
1360|779		1361	.... (*c*)
419|2010		419	
408|2337		408	.... (*d*)
10|96730		11	
9|07186		9	.... (*e*)
1|89544		2	

For the method of obtaining lines *(a)*, *(b)*, *(c)*, *(d)* and *(e)* see the preceding Example 1.

If the 'final remainder' be greater than half of the last contracted divisor increase the last figure of the quotient by 1. The quotient will now be *in excess* but the error will be less than if the quotient had been left in defect.

The methods of contraction exhibited in the two preceding examples may easily be adapted to calculations in which the result is required to be correct to a given number of decimal places. Exercises of this character will arise later on in the calculations of Interest and Discount. It should, however, be particularly noted that asking for a calculation to be carried to the sixth or any other specified place of decimals without indicating the unit to be employed gives no idea of the degree of approximation required.

EXERCISE LXXXII.

1. If the length of the polar diameter of the earth be 41,710,000 ft., correct to 5 significant figures, between what limits must the correct length lie? About what degree of approximation is obtained by taking 41,710,000 ft. as the correct length?

2. If the mean equatorial radius of the earth be 3963·3 miles correct to 5 significant figures, about what degree of approximation is obtained by taking this as the correct length?

3. According to Regnault a given volume of mercury at 0°C is from 13·59578 to 13·59602 times as heavy as an equal volume of water at 0°C. Accepting these numbers, to how many significant figures do they determine the ratio of the weights of equal volumes of the liquids both at 0°C and what is the ratio as thus determined?

4. Volkmann has given 13·5953 to 1 as the ratio of the weights of equal volumes of mercury and water both at 0°C, other investigators have obtained a ratio as high as 13·603 to 1; including these with Regnault's values, to how many significant figures has the ratio been determined, and what is the ratio as thus determined? To about what degree of approximation has the ratio been determined?

Find the value of the following, correct in each case to 6 significant figures:

5. $4{\cdot}78534 + 2{\cdot}87496 + 1{\cdot}63 + 0{\cdot}0847521.$

6. $0{\cdot}25 + 0{\cdot}03125 + 0{\cdot}015625 + 0{\cdot}00390625 + 0{\cdot}000244141 + 0{\cdot}000004604.$

7. $3{\cdot}16227766 + 1{\cdot}77827941 + 1{\cdot}33352143 + 1{\cdot}15478198 + 1{\cdot}0746083 + 1{\cdot}03663293.$

8. $72{\cdot}836 + 4{\cdot}598978 + 98{\cdot}347223 + 403{\cdot}8.$

9. $4{\cdot}7856 + 2{\cdot}8937584 + 0{\cdot}3796487 - 7{\cdot}78536893.$

10. $985748 + 286949 + 859374 + 586207 - 43625.$

11. $28{\cdot}3160 \times 2{\cdot}20462.$

12. $277{\cdot}274 \times 252{\cdot}458 \div 7000.$

13. $65{\cdot}582 \times 65{\cdot}582.$

14. $87{\cdot}51 \times 0{\cdot}8751.$

15. $8{\cdot}467 \times 8{\cdot}467 \times 8{\cdot}467.$

16. $88{\cdot}7904 \times 8{\cdot}87904 \times 0{\cdot}887904.$

17. $0{\cdot}7065 \times 0{\cdot}7065 \times 0{\cdot}7065.$

18. $0{\cdot}7937 \times 0{\cdot}7937 \times 79{\cdot}37.$

19. $0{\cdot}8583 \times 8583 \times 85{\cdot}83.$

20. $5977 \times 5977 \times 5977$

21. $100 \div 39{\cdot}3704.$

22. $1 \div 0{\cdot}220462.$

23. $34902 \div 3{\cdot}141593$.

24. $1 \div 3141593$.

25. $1 \div 43429448$.

26. $3{\cdot}162278 \div 1{\cdot}77828$.

27. $4579 \div 4187$.

28. $81350 \div 50549$.

29. $92174 \div 227771$.

30. $33026 \div 37493$.

A metre = 39·37043 and a kilogramme = 2.204621 lb. both to 7 significant figures. A decimetre is the tenth part of a metre and a litre is a cubic decimetre.

31. How many kilogrammes are equal to a ton?

32. How many kilogrammes are there in an Imperial or "long" ton (2240 lb.)?

33. How many metres are there in a mile?

34. How many square yards are there in 100 square metres?

35. How many hundred square metres would make an acre?

36. How many cubic decimetres are there in a cubic foot?

37. A litre of water at 60°F weighs 2·2021444 lb. and a gallon of water at the same temperature weighs 10 lb. How many litres are there in a gallon?

38. What is the weight of a cubic foot of water at 60°F?

39. How many cubic inches are there in a gallon?

40. How many feet are there in the diameter of a circle whose circumference measures forty million metres? (Circumference = radius $\times 3{\cdot}14159$.)

41. Find the product of 2·9957323 and ·4342945 correct to 6 decimal places.

42. Find the value of $5{\cdot}7037825 \times {\cdot}4342945$ correct to 6 decimal places.

43. Find the value of $3{\cdot}14159265 \times {\cdot}96 \times {\cdot}995 \times {\cdot}9998 \times {\cdot}99992 \times {\cdot}999997$, correct to 6 decimal places?

44. Find the value of $2{\cdot}7182818 \times {\cdot}8 \times {\cdot}992 \times {\cdot}9993 \times .99998 \times {\cdot}999993 \times {\cdot}9999994$ correct to 7 decimal places.

45. Find the value of $2{\cdot}3025851 \times {\cdot}9 \times {\cdot}97 \times {\cdot}995 \times {\cdot}99995 \times {\cdot}999997$ correct to 7 decimal places.

46. Find the value of $3{\cdot}1416 \times 3{\cdot}1416 \times 3{\cdot}1416$ to the nearest integer.

II. INVOLUTION.

The *square* of a number is the product of two factors each equal to the number.

The square of 3 is $3\times3=9$; the square of 5 is $5\times5=25$.

The *cube* of a number is the product of three factors each equal to the number.

The cube of 2 is $2\times2\times2=8$; the cube of 5 is $5\times5\times5=125$.

The *fourth*, *fifth*, *sixth*,.....*power* of a number is the product of four, five, six,.....factors each equal to the number.

The fourth power of 7 is $7\times7\times7\times7=2401$.

The number itself is the first power, the square is the second power and the cube is the third power of the number.

The *base* of a power is the number used as a factor in forming the power.

The *exponent* or *index of a power* is the number which expresses how often the base occurs as factor in forming the power. The figures of an exponent are usually made somewhat smaller than those of its base and are placed on the right of the base and a little above it.

6×6 is written 6^2 which is read "6 square." Here 6 is the base and 2 is the exponent. $10\times10\times10\times10$ is written 10^4 which is read "10 to the fourth," *power* being understood after *fourth*. Here 10 is the base and 4 is the exponent.

EXERCISE LXXXIII.

Write the following products as powers :—

1. 3×3.
2. $2\times2\times2$.
3. $5\times5\times5\times5$.
4. $4\times4\times4\times4\times4$.
5. $1{\cdot}5\times1{\cdot}5\times1{\cdot}5\times1{\cdot}5\times1{\cdot}5$.
6. $0.1\times0{\cdot}1\times0{\cdot}1\times0{\cdot}1$.
7. $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$.
8. $\frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}$.

Write the following powers as products and compute their values :—

9. 6^2.
10. 2^6.
11. 3^4.
12. 4^3.
13. 2451^2.
14. 924^3.
15. 273^4.
16. 2^{10}.
17. $1{\cdot}1^3$.
18. $1{\cdot}02^3$.
19. $0{\cdot}1^5$.
20. $0{\cdot}01^5$.
21. 49^2.
22. $4{\cdot}9^2$.
23. $0{\cdot}49^2$.
24. 530^3.
25. 53^3.
26. $5{\cdot}3^3$.
27. $0{\cdot}53^3$.
28. $(\frac{1}{2})^6$.
29. $(\frac{2}{3})^5$.
30. $(\frac{5}{7})^4$.
31. $\frac{5^4}{7^4}$.
32. $3^2 \times 5^3$.
33. $2^5 \times 5^3 \times 7 \times 11^4 \times 13^2$.
34. $2^{10} \div 10^2$.

Resolve the following numbers into their prime factors, expressing the repetition of a factor by an exponent :—

35. 360. *36.* 70200. *37.* 11368. *38.* 1024. *39.* 5837832.

Prove the truth of the following statements :—

40. $(2^3)^4 = 2^{12}$.
41. $(3^4)^3 = 3^{12}$.
42. $(5^4)^3 = 5^{12}$.
43. $(2^5)^4 = 2^{20}$.
44. $(10^3)^7 = 10^{21}$.
45. $\{(\frac{1}{2})^3\}^5 = (\frac{1}{2})^{15}$.
46. $2^3 \times 2^4 = 2^{3+4} = 2^7$.
47. $3^4 \times 3^3 = 3^{4+3} = 3^7$.
48. $0{\cdot}5^3 \times 0{\cdot}5^4 = 0{\cdot}5^7$.
49. $2^7 \times 2^5 = 2^{12}$.
50. $10^{10} \times 10^5 = 10^{15}$.
51. $(\frac{2}{3})^3 \times (\frac{2}{3})^5 = (\frac{2}{3})^8$.
52. $2^7 \div 2^3 = 2^{7-3}$.
53. $0{\cdot}2^5 \div 0{\cdot}2^2 = 0{\cdot}2^3$.
54. $10^{10} \div 10^7 = 10^3$.
55. $(\frac{3}{4})^6 \div (\frac{3}{4})^2 = (\frac{3}{4})^4$.
56. $3^2 \times 4^2 = (3 \times 4)^2 = 12^2$.
57. $5^3 \times 3^3 = 15^3$.
58. $4^4 \times 5^4 = 20^4$.
59. $3^5 \times 10^5 = 30^5$.
60. $3^4 \times (\frac{1}{2})^4 = (\frac{3}{2})^4$.
61. $25^2 = 5^4$.
62. $2^{12} = 4^6 = 8^4$.
63. $72^2 = 2^6 \times 3^4 = 2^2 \times 6^4$.

Find the value, correct to 4 significant figures of

64. $\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \frac{1}{5^5} + \frac{1}{5^6} + \frac{1}{5^7} + \frac{1}{5^8} + \frac{1}{5^9}$.

65. $\frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \frac{1}{6^4} + \frac{1}{6^5} + \frac{1}{6^6} + \frac{1}{6^7} + \frac{1}{6^8} + \frac{1}{6^9}$.

66. $\frac{1}{9} - \frac{1}{9^2} + \frac{1}{9^3} - \frac{1}{9^4} + \frac{1}{9^5} - \frac{1}{9^6} + \frac{1}{9^7}$.

67. Show that \$100 at interest for 4 years at 5% compounded annually will amount to $\$100 \times 1{\cdot}05^4$.

68. \$320 at interest for 6 years at 4% compounded annually will amount to $\$320 \times 1{\cdot}04^6$.

69. \$125 at interest for 4 years at 6% compounded semi-annually will amount to $\$125 \times 1{\cdot}03^8$.

70. \$1000 at interest for 3 years at 6% compounded quarterly will amount to $\$1000 \times 1{\cdot}015^{12}$.

Express by power notation the amount of \$1 at interest for 3 years at 5% compounded

71. Yearly. *72.* Half-yearly. *73.* Quarterly.

74. Show that $\$1 \div 1{\cdot}04^3$ is the present worth of \$1 due 3 years hence, discounted at 4% *interest*, compounded annually.

Express by power-notation the present value of:

75. \$1000 due 5 years hence discounted at 4% interest, compounded annually.

76. \$500 payable $7\frac{1}{2}$ years hence discounted at 4% interest, compounded semi-annually.

77. \$500 payable $5\frac{1}{2}$ years hence discounted at 4% interest, compounded quarterly.

Find the discount off \$125 payable 5 years hence at

78. 5% *discount.* *79.* 5% *simple interest.*

80. 5% *interest* compounded annually.

81. 5% *interest* compounded semi-annually.

82. J. S. deposits \$100 yearly in a savings bank and receives 3% interest on it which he allows to remain on deposit; express by power-notation the amount to his credit when he has made his 3rd, 4th, 5th and 6th deposits respectively, and compute the amount in each case to the nearest cent.

83. Show that the present value of a bond for \$1000 payable 10 years hence and bearing interest at 4% payable annually, money being worth 5%, is

$$\$40\left\{\frac{1}{1{\cdot}05}+\frac{1}{1{\cdot}05^2}+\frac{1}{1{\cdot}05^3}+\frac{1}{1{\cdot}05^4}+\frac{1}{1{\cdot}05^5}+\frac{1}{1{\cdot}05^6}+\ldots+\frac{1}{1{\cdot}05^{10}}\right\}+\frac{\$1000}{1{\cdot}05^{10}}.$$

and compute this value to the nearest dollar.

III. EVOLUTION.

A number is called the *square root* of its square, the *cube root* of its cube, the *square root* of its fourth power, etc.

Ex.—Number 1, 4, 9, 16, 25, 36, 49, 64, 81, 1,00, 4,00, 9,00.
Square Root 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30.

Number 1, 8, 27, 64, 125, 216, 343, 512, 729, 1,000.
Cube Root 1 2 3 4 5 6 7 8 9 10.

The square root of a number is denoted by prefixing the symbol $\surd$ to the number, the cube root by prefixing $\sqrt[3]{}$, the fourth root by prefixing $\sqrt[4]{}$, and all the other roots are similarily denoted.

Ex.— $\surd 9 = 3$, $\surd 9{,}00 = 30$, $\surd 9{,}00{,}00 = 300$, $\sqrt[3]{125} = 5$ $\sqrt[3]{125{,}000} = 50$, $\sqrt[5]{32} = 2$.

EXERCISE LXXXIV.

Prove that

1. $\surd 1{,}44 = 12$.
2. $\surd 1{,}44{,}00 = 120$.
3. $\surd 1{,}44{,}00{,}00 = 1200$.
4. $\surd 1{\cdot}44 = 1{\cdot}2$.
5. $\surd 0{\cdot}01{,}44 = 0{\cdot}12$.
6. $\surd 0{\cdot}00{,}01{,}44 = 0{\cdot}012$.
7. $\surd 289 = 17$.
8. $\surd 0{\cdot}25 = 0{\cdot}5$.
9. $\surd 2{,}25 = 15$.
10. $\surd 2{\cdot}25 = 1{\cdot}5$.
11. $\surd 0{\cdot}02{,}25 = 0{\cdot}15$.
12. $\surd 56{,}25 = 75$.
13. $\surd 56{\cdot}25 = 7{\cdot}5$.
14. $\surd \frac{1}{4} = \frac{1}{2}$.
15. $\surd \frac{4}{9} = \frac{2}{3}$.
16. $\surd \frac{16}{81} = \frac{4}{9}$.
17. $\surd \frac{256}{2401} = \frac{16}{49}$.
18. $\sqrt[3]{1728} = 12$.
19. $\sqrt[3]{1{\cdot}728} = 1{\cdot}2$.
20. $\sqrt[3]{\frac{8}{125}} = \frac{2}{5}$.
21. $\sqrt[4]{81} = 3$.

In extracting the root of any number note :

1° That after obtaining the left hand figure of the root each additional figure affixed on the right in the root gives two additional figures on the right in the square of that root, three on the right in the cube, four on the right in the fourth power, etc.

2° That for each figure on the right of the decimal point in the root, there will be two figures on the right of the decimal point in the square of that root, three on the right in the cube, four on the right in the fourth power, etc.

3° That if a be near to the square root, b to the cube root, c to the fourth root, etc., of the number N, then will $\frac{N+a^2}{2a}$ be nearer than a to the square root, $\frac{N+2b^3}{3b^2}$ be nearer than b to the cube root, $\frac{N+3c^4}{4c^3}$ be nearer than c to the fourth root, etc.

Hence in preparing to extract any root of a number—

1° Begin at the decimal point and mark off the figures left and right in pairs in case of the square root, in sets of three in case of the cube root, in sets of four in case of the fourth root, etc.

2° If the figures on the right of the decimal point do not give an exact number of sets, they must be made to do so by affixing one or more zeros on the right of the number.

3° If an approximate root r *be found of the number* N, *this root must be corrected by adding to it* $\frac{N-r^2}{2r}$ *in the case of the square root,* $\frac{N-r^3}{3r^2}$ *in case of the cube root,* $\frac{N-r^4}{4r^3}$ *in the case of the fourth root, etc.* (It is usual to take only the first digit of this *correction*, at least for the first four or five corrections.)

[In the case of the square root, if a be an approximate root and h be the correction of a, the next correction, that of $a+h$, will be

$$\frac{N-(a+h)^2}{2(a+h)} \text{ which may be written } \frac{(N-a^2)-(2a+h)h}{2(a+h)};$$

and in this $N-a^2$ has already been computed when finding the correction h.]

EXERCISE LXXXV.

Find the square root of :—

1. 484.
2. 9801.
3. 1849.
4. 103041.
5. 1030·41.
6. 10·3041.
7. 2321·3124.
8. 0·0050367409.
9. 0·07001316.

EXERCISE LXXXVI.

Find, correct to five significant figures, the square root of:

1. 550184000.
2. 1281.
3. 7·46.
4. 74·49.
5. 0·3131.
6. 0·062.
7. 5.
8. 500.
9. 50000.
10. 0·05.
11. 0·0005.
12. 50.
13. 5000.
14. 500000.
15. 0·5.
16. 0·005.
17. 0·00005.
18. 0·4.
19. 5×4.
20. $5 \times 4 \times 9$.
21. 8×10.
22. $1 \times 2 \times 3 \times 4 \times 5 \times 6$.
23. $5 \div 4$.
24. $20 \div 49$.
25. $\frac{3}{4}$.
26. $\frac{5}{8} = \frac{10}{16}$.
27. $\frac{7}{11}$.

Compute the square roots of 2, 3, 6, 12 and 18 to five significant figures each and use the results to prove that:—

28. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$.
29. $\sqrt{6} \times \sqrt{2} = \sqrt{12} = 2\sqrt{3}$.
30. $\sqrt{6} \times \sqrt{3} = \sqrt{18} = 3\sqrt{2}$.
31. $\sqrt{2} \times \sqrt{3} \times \sqrt{6} = \sqrt{36} = 6$.
32. $\sqrt{2} - 1$ is the reciprocal of $\sqrt{2} + 1$.
33. $\sqrt{3} - \sqrt{2}$ " " " $\sqrt{3} + \sqrt{2}$.
34. $2 - \sqrt{3}$ " " " $2 + \sqrt{3}$.
35. $3 - 2\sqrt{2}$ " " " $3 + 2\sqrt{2}$.
36. $5 - 2\sqrt{6}$ " " " $5 + 2\sqrt{6}$.
37. $7 - 2\sqrt{12}$ " " " $7 + 2\sqrt{12}$.

Compute to three significant figures the cube root of :--

38. 10. *39.* 220. *40.* 264·61.

CHAPTER XI.

MENSURATION.

A. The sides about the equal angles of mutually equiangular triangles are proportionals and the corresponding sides are those opposite equal angles. (*Euclid vi, 4.*)

B. The square on the hypothenuse of a right-angled triangle is equal to the sum of the squares on the other sides. (*Euclid i, 47.*)

C. Circle. The Circumference $= 3\frac{1}{7}$ diameters, nearly.

[The circumference falls short of $3\frac{1}{7}$ diameters at a rate which is somewhat less than an inch in 200 feet; it is shorter than 3·1416 diameters by nearly an inch in every 6 miles and is less than $\frac{355}{113}$ diameters by not quite an inch in every 165 miles. The approximation $3\frac{1}{7}$ diameters is quite close enough for *ordinary* practical purposes, the errors of measurement generally exceeding the error of computation arising from using this approximation.]

D. Ellipse with nearly equal axes. The perimeter is nearly $3\frac{1}{7}$ times the side of the square which is double the sum of the squares on the semi-axes.

i. *Rectangles.* The numeric of area is the product of the numerics of the lengths of two adjacent sides.

ii. *Parallelogram.* The numeric of the area is the product of the numerics of the altitude and the length of the base.

iii. *Triangle.* The numeric of the area is *one-half* of the product of the numerics of the altitude and the length of the base. It is also the square root of the continued product of the numerics of the semi-perimeter and the three remainders obtained by substracting the length of each side separately from the semi-perimeter.

iv. *Trapezoid.* The numeric of the area is *one-half* of the product of the numerics of the altitude and the sum of the lengths of the parallel sides.

v. *Circle.* The area is nearly $3\frac{1}{7}$ times that of the square on the radius.

vi. *Sector of Circle.* The numeric of the area is *one-half* of the product of the numerics of the lengths of the radius and the bounding arc.

vii. *Ellipse.* The area is nearly $3\frac{1}{7}$ times that of the rectangle under the semiaxes.

viii. *Mantel of Right Cylinder.* The numeric of the area is the product of the numerics of the altitude and the perimeter of the base.

ix. *Mantel of Right Circular Cone.* The numeric of the area is *one-half* of the product of the numerics of the slant height and the perimeter of the base.

x. *Sphere.* The area of the surface is nearly $3\frac{1}{7}$ times that of the square on the diameter.

xi. *Cap or Zone of Sphere.* The area is nearly $3\frac{1}{7}$ times that of the rectangle under the diameter of the sphere and the altitude of the cap or zone.

xii. The areas of *Similar Figures* or of *Similar Surfaces* are to one another as the areas of the squares on corresponding lines of the figures or of the surfaces.

I. *Quad* or *Parallelepiped.* The numeric of the volume is the product of the numerics of the lengths of three adjacent edges.

II. *Prism.* The numeric of the volume is the product of the numerics of the altitude and the area of the base.

III. *Pyramid.* The numeric of the volume is *one-third* of the product of the numerics of the altitude and the area of the base.

IV. *Wedge.* The numeric of the volume is *one-sixth* of the continued product of the numerics of the altitude, the width of the base and the sum of the lengths of the three parallel edges.

V. *Prismatoid.* The numeric of the volume is *one-sixth* of the product of the numerics of the altitude and the sum obtained by adding the areas of the parallel faces to four times the area of the midcross-section. (PRISMOIDAL FORMULA.)

VI. *Cylinder.* The numeric of the volume is the product of the numerics of the altitude and the area of the base.

VII. *Cone.* The numeric of the volume is *one-third* of the product of the numerics of the altitude and the area of the base.

VIII. *Frustum of Pyramid* or of *Cone.* The numeric of the volume is *one-third* of the product of the numeric of the altitude and the sum obtained by adding the numerics of the areas of the two parallel faces to the square root of the product of these two numerics.

IX. *Sphere.* The volume is *two-thirds* of the volume of the circumscribing cylinder. It is also nearly *four-thirds* of $3\frac{1}{7}$ times the volume of the cube on the radius.

X. *Sector of Sphere.* The numeric of the volume is *one-third* of the product of the numerics of the radius and the area of the cap or zone.

XI. *Ellipsoid.* The volume is nearly *four-thirds* of $3\frac{1}{7}$ times that of the quad under the semiaxes.

XII. The volumes of *Similar Solids* are to one another as the volumes of the cubes on corresponding lines of the solids.

A cubic foot of water at 52°F. weighs 62·4 lb.

A cubic foot $= (25 - \frac{1}{25})$ quarts.

EXERCISE LXXXVII.

1. A stick 3′ in length placed upright on the ground is found to cast a shadow 2′ 6″ long, what must be the height of a flagpole which casts a shadow 28′ in length?

2. The parallel sides of a trapezoid are respectively 27 ft. and 35 ft. in length and the non-parallel sides are respectively 18 ft. 7 in. and 23 ft. 11 in. long. The latter sides are produced to meet; find the respective lengths of the produced sides between the point of meeting and the shorter of the parallel sides of the trapezoid.

3. Taking the diameter of the sun to be 880,000 miles and the sun's distance from the earth to be 92,400,000 miles, what must be the diameter of a circular disk that it may just hide the sun when held between the eye and the sun and 21 inches in front of the eye?

4. Three men A, B and C stand in a row on a level pavement, A's height is 5′ 3½″, B's is 5′ 9″ and C's is 6′ 1½″; if A stand 10′ to the right of B, how far to the left of B must C stand that the tops of the heads of the three men may range in a straight line?

5. How many yards of fencing-wire will be required to enclose a rectangular field thrice as long as it is wide, if the field contain 10 acres and the fence be made 5 wires high?

6. A six-foot sidewalk of 3 in. planks is to be laid around a rectangle 266 ft. 8 in. by 480 ft., the inner edge of the sidewalk to be twelve inches out from the sides of the rectangle. Find the value at $14 the M, board-measure, of the planking for the sidewalk.

7. Find the areas of the outer and the inner surface of a hollow iron cube measuring 8 in. on the outside edge, the iron being ⅜ in. thick.

8. The length of the base of a parallelogram is 45 ft.; the length of the perpendicular on the base from the opposite side is 28 ft.; the length of a side adjacent to the base is 35 ft.; find the length of the perpendicular on this side from the side opposite to it.

9. The adjacent sides of a parallelogram measure 132 ft. and 84 ft. respectively and the area of the parallelogram is two-thirds of that of a square of equal perimeter. Find the perpendicular distance between each pair of parallel sides.

10. Find the area of a quadrilateral one of whose sides measures 23·29 chains and the perpendiculars on this side from the opposite angles of the quadrilateral 17·75 chains and 13·45 chains respectively, the distances of the feet of these perpendiculars from the adjacent angles being 3·64 chains and 2·40 chains respectively.

11. The lengths of the parallel sides of a trapezoid are 12 ft. and 17 ft. and the perpendicular distance between these sides is 8 ft. A straight line is drawn across the trapezoid parallel to the parallel sides and midway between them. Find the areas of the two parts into which the trapezoid is thus divided.

12. The area of a triangle is 551 sq. yd. and the length of its base is 95 ft. Two straight lines are drawn across the triangle parallel to the base, dividing into three equal parts the perpendicular from the vertex on the base. Find the areas of the parts into which the triangle is thus divided.

13. The lengths of the parallel sides of a trapezoidal field are 15·80 chains and 18.70 chains respectively and the perpendicular distance between these parallel sides is 14·40 chains. Four straight lines are drawn across the field parallel to the two parallel sides and dividing the distance between these sides into five equal parts. Find the areas of these five parts of the field.

14. A rectangular field 200 yd. long is surrounded by a road of the uniform width of 60 ft. The total area of both field and road is 9 A. 1240 sq. yd. Find the width of the field.

15. A rectangular field 780 ft. in length is surrounded by a road of the uniform width of 50 ft., the area of the whole road being 15000 sq. yd. Find the area of the field.

16. Find to the nearest gallon the capacity of an open quadrate tank measuring 7′ 6″ by 6′ 4″ by 5′ 8″ externally; the material of which the tank is made being $1\frac{1}{4}$ inches in thickness.

17. Three cubes of lead measuring respectively $\frac{1}{2}$, $\frac{3}{8}$ and $\frac{5}{8}$ of an inch on the edge were melted together and cast into a single cube. Find the length of the edge of the cube thus formed, neglecting loss of lead in melting and casting.

18. Four cubes of lead measuring respectively 6, 7, 8 and 9 inches on the edge were melted together and cast into a single cube. Find the length of the edge of the cube thus formed, if 4 per cent. of the lead was lost in the melting and casting.

19. Find the volume of a cube the area of whose surface is 100·86 sq. in.

20. The surface of a cube measures 30 sq. in. Find the area of the surface of a cube of five times the volume of the former.

21. The volume of a cube is 30 cu. in. Find the volume of a cube whose surface has an area five times the area of the surface of the former cube.

22. A cube measures 5 in. on the edge. A second cube is of thrice the volume of the first. By how much does the length of an edge of the second cube exceed that of an edge of the first cube?

23. A cube measures 5 in. on the edge. Find the volume of a cube whose surface-area is thrice that of the former cube.

24. A quadrate cistern is 5 ft. wide by 6 ft. long by 4 ft. 2 in. deep. Its width and its length are each increased by 6 inches. How much deeper must it be made that the total increase of its capacity may be 250 gallons?

25. The height of a solid six-inch cube of India-rubber is diminished by pressure to 5·85 in. If the volume of the solid remain the same and the lateral expansion be uniform throughout, what will be the dimensions of the new base?

26. Find the weight of the air in a rectangular room measuring 27′ 8″ by 23′ 5″ by 12′ 4″, the weight of the air being the ·001295 of the weight of an equal volume of water.

27. If a cubic foot of gold weigh 1200 lb., find the thickness of gold-leaf of which 1200 leaves $3\frac{1}{2}$ inches square weigh an ounce troy.

28. What will be the weight of 36 iron rods each 14 ft. long and of cross section $\frac{3}{8}$ of an inch square, if the specific gravity of the iron be 7·7?

29. What weight will just keep under water a stick of square-timber measuring 36 ft. by 10 inches square, the specific gravity of the wood being ·725?

30. An open quadrate tank is 5 ft. 6 in. long, 4 ft. 3 in. wide and 3 ft. 8 in. high, the sides and bottom of the tank are $\frac{3}{4}$ in. thick. Find the number of cubic inches of material in the vessel.

31. The length of a prism is 9 ft. 4 in. A right cross-section of the prism is a quadrilateral, one of whose diagonals measures 3 ft. 7 in. and the perpendiculars on that diagonal from the opposite angles of the section are respectively 1 ft. 5 in. and 1 ft. 7 in. long Find the volume of the prism.

32. The fabled wall of China was said to be 25 ft. wide at the bottom, 15 ft. wide at the top, 20 ft. high and 1500 miles long. How many cubic yards of material would such a wall contain?

33. How many cubic yards of earth must be removed in the digging of a ditch 147 ft. long, 8 ft. wide at the top, 5 ft. wide at the bottom and 4 ft. 6 in. deep, the ends of the ditch being vertical?

34. How many gallons of water will fill a horse trough 7 ft. 6 in. long, 10 in. deep, 14 in. wide at the top and 11 in. wide at the bottom; the ends of the trough being at right angles to the bottom and sides?

35. A stream flows at the rate of three miles per hour through a trough whose cross-section is a trapezoid. The width of the bottom of the trough is 21 inches, the depth of the water is 4·5 inches and the width of the surface of the water in the trough is 25 inches. How many gallons flow through the trough per minute?

36. A school-room with attic ceiling is 32 ft. long by 28 ft. wide. The ceiling at the side walls is 10 ft. above the floor and slopes upward until it attains a height of 14 ft. 6 in. and then becomes level, the width of the level part being 12 ft. The ceiling meets the end walls at right angles. Find the air-capacity of the school-room.

37. The Great Pyramid of Egypt when complete was 480 ft. 9 in. in height, and its base was a square 764 ft. in length; in its present condition the pyramid is 450 ft. 9 in. high and its base is a square 746 ft. long and wide. Find to the nearest cubic yard the volume of the pyramid in its complete and also in its present state.

38. The representative gold pyramid in the International Exhibition of 1862 was 10 ft. square at the base and 44 ft. 9¼ in. high. Find the volume of the pyramid, and the weight and the value of the gold represented by it, taking the specific gravity of the gold at 19·25 and its value at $20·67 per ounce troy.

39. Find the capacity of a cart the top of which measures 4′ 3″ by 3′ 8″; the bottom, 3′ 9″ by 3′ 2″; and the depth, 2′ 3″.

40. How many gallons of water will fill a ditch 2 ft. deep, the top and bottom of the ditch being rectangles whose corresponding dimensions are 148 ft. by 3 ft. 4 in. and 146 ft. 6 in. by 2 ft. 3 in.?

41. Find the weight of an iron shaft whose ends are rectangles, one end measuring 10·5 in. by 17 in., the other end measuring 7 in. by 12 in., the length of the shaft being 13 ft. 6 in. and the specific gravity of the iron 7·7.

42. What weight will just sink a scow in the form of a hollow prismoid with rectangular base, the length of the scow over all being 14 ft. 1 in.; its width, 3 ft. 8 in.; its full depth, 2 ft. 11 in.; the length of the bottom outside, 12 ft.; the width of the bottom 3 ft. and its weight 920 lb.

43. Find the volume of a pile of broken stones, the base of the pile being a rectangle measuring 13 ft. 6 in. by 7 ft. 5 in.; the top of the pile a rectangle measuring 12 ft. 2 in. by 6 ft.; and the height of the pile being 2 ft. 10 in.

44. It is usual to take as the measure of the volume of a pile of broken stones the product of the measure of the altitude of the pile and the measure of the area of its midcross-section. By how much would the volume thus calculated be in defect of the actual volume in the case of the pile described in the problem immediately preceding.

45. The base of a wedge is a rectangle measuring 3·6 in. by 2·4 in., the length of the opposite edge is 3 in., the height of the wedge is 8 in. Find the volume 1° if the three-inch edge is parallel to the longer side of the base; 2° if it is parallel to the shorter side of the base.

46. Find the number of cubic feet in a stick of square timber 18″ square at one end, 14″ square at the other end and 36′ long.

47. Find the weight of a frustum of a square pyramid of marble, the height of the frustum being 6 ft. 6 in.; the length of an edge of the base, 4 ft. 4 in., and of an edge of the top, 2 ft. 8 in., the weight of a cubic foot of marble 172 lb.

48. Find the length of the hypothenuse of a right-angled triangle, the lengths of the other sides being 777 in. and 464 in.

49. Find the length of the diagonal of a square whose side is one foot long.

50. Find the length of the diagonal of a cube whose edge is one foot long.

51. What is the length of the edge of the largest cube that can be cut out of a sphere 6 inches in diameter?

52. A quad measures 24 in. by 11·7 in. by 4·4 in. Find the lengths of the diagonals of its faces.

53. A quad is 14 ft. long by 5 ft. wide by 2 ft. thick. Find the length of its diagonal.

54. The diagonals of the faces of a quad are respectively 25 in., 23·79 in. and 9·79 in long. Determine the lengths of the edges of the quad.

Find the areas of the triangles the lengths of whose sides are respectively

55. 13 yd., 10 yd. and 13 yd.

56. 13 yd., 24 yd. and 13 yd.

57. 1.23 ch., 5·95 ch. and 6·76 ch.

58. 73·2 ch., 45·5 ch. and 87·6 ch.

59. What will be the value at $73 per acre of a triangular piece of land the lengths of whose sides are 478·5 chains, 329·6 chains and 237·4 chains respectively?

60. The lengths of the sides of a triangle are respectively 212 ft., 225 ft. and 247 ft. A straight line is drawn across the triangle joining the mid-points of two of the sides. Find the area of the trapezoid thus formed.

61. The minute hand of a clock measures 1 ft. $3\frac{3}{4}$ in. from the centre of its arbor to the tip of the hand. Find the distance travelled by the tip of the hand during the course of 365 days.

62. Find the length of the radius of a wheel which made 1600 revolutions in rolling 3·25 miles.

63. A circular path is 400 yd. in length on its inner edge. What will be its length 5 ft. out from that edge all around?

64. Assuming that the earth every 365 da. 6 hr. 9 min. 9 sec. describes around the sun a circle of 92,890,000 miles radius, find the average speed per hour of the earth in this path round the sun.

65. How many revolutions per min. does a wheel 5′ 6½″ in diameter make if it is travelling at the rate of 27½ mi. per hour?

66. A locomotive wheel 4′ 4½″ in diameter, making an average of 767 revolutions per 5 min., travels for 4 hr. 10 min. How far does it go in that time?

67. The front and hind wheels of a waggon being 3 ft. 8 in. and 4 ft. 2 in. respectively in diameter, how many revolutions will a front wheel make more than a hind wheel for every mile travelled?

68. The lengths of the diameters of the front and hind wheels of a carriage being 3 ft. 4 in. and 4 ft. respectively, how far will the carriage have to travel before the front wheel will have made 100 revolutions more than the hind wheel?

69. Find the length of the equator; 1°, assuming it to be an ellipse the lengths of whose semiaxes are 20,926,629 feet and 20,925,105 feet respectively; 2°, assuming it to be a circle of 20,926,202 feet radius.

70. The French metre was originally defined to be the 10,000,000th part of the length of a meridian quadrant taken from the equator to the pole. Had this definition been retained what would be the length of a meter in inches, if the length of the polar axis of the earth be 41,709,790 ft. and the length of the equatorial diameter be 41,852,404 ft.

71. Find the area of a circle of 2½ in. radius.

72. Find the area of a circle of ½ in. in diameter.

73. The circumference of the circular basin of a fountain measures 117·81 ft. on the outside of the masonry and the thickness of the masonry is 30 in. Find the area of the surface of the water within the basin.

74. A cent is an inch and a penny is $\frac{1}{10}$ ft. in diameter. If a cent lie wholly on top of a penny, find the area of the upper surface of the penny remaining uncovered.

75. A rectangular room, 27′ 6″ by 13′ 6″, has a semicircular bow-window 8′ 4″ in diameter, thrown out at the side. Find the area of the floor of the whole room.

76. The perimeters of a circle, a square and an equilateral triangle are each 6 ft. in length. Find by how much the area of the circle exceeds the area of each of the other figures.

77. A circular fish-pond whose area is 2·5 acres is surrounded by a walk 3 yd. wide. Find the cost at 9 ct. per square yard of gravelling the walk from its outer boundary to within one foot of the edge of the pond.

78. Three circles of 2 ft. radius each, touch each other. Find the area of the figure enclosed by them.

79. An elliptic flower-bed is described by means of a string 16 ft. long passing over two pegs 6 ft. apart. What is the area of the bed?

80. A lawn in the shape of an ellipse the lengths of whose axes are 98 ft. and 58 ft. is surrounded by a walk 2 yards wide. Find the area of the walk.

81. The length of a cylinder is 22 ft. and its least girth is 22 in. Find the area of the mantel.

82. Find the area of a right circular cylinder of 25 in. altitude and 12 in. radius of base.

83. Find the area of the whole surface of a right circular cylinder of 15 in. radius and 5 ft. altitude.

84. Find the area of the *whole* surface of a cylindric pipe 8 ft. 6 in. long and an inch and a quarter thick, the length of the internal diameter being $10\frac{1}{2}$ in.

85. Find the area of the mantel of a right circular cone whose dimensions are, slant height 4 ft. 6 in., length of radius of base 1 ft. 3 in.

86. How many yards of canvas 32 in wide will be required to make a conical tent 15 ft. wide and 10 ft. high, if 10% of the canvas is cut away or turned in, in the making of the tent.

87. Find the area of the surface of a sphere of 3 in. radius.

88. Find the area of the surface of a sphere 12 inches in circumference.

89. Find in square miles the area of the surface of the earth assuming it to be practically an oblate spheroid the lengths of whose semiaxes are 20,926,202 feet and 20,854,895 feet respectively.

90. The length of the radius of the base of a right circular cylinder is 5 in. and the altitude of the cylinder is 8 in. Find its volume.

91. Find the content of a cylindrical measure 8 in. deep by $10\frac{1}{2}$ in. diameter.

92. Find the content in gallons of a cylindrical measure $10\frac{1}{2}$ in. deep by 8 in. diameter, taking 25 qt. to the cu. ft.

93. A cubic foot of brass is drawn into wire the twentieth of an inch in diameter. Find the length of the wire.

94. Mr. C. V. Boys has drawn quartz fibres which have been estimated to be only the millionth of an inch in diameter. How many miles of such a fibre would a grain of sand make, the grain being a right circular cylinder one-hundredth of an inch long by one-hundredth of an inch in diameter.

95. Find the thickness of the lead in a pipe of three-quarter inch bore, if 10 ft. of the pipe weigh 21 lb. and a cubic foot of lead weigh 712 lb.

96. Find the volume of a cone whose slant height is 65 in. and whose base is a circle 32 in. in diameter.

97. How many gallons will a hemispherical bowl 18 inches in diameter hold?

98. Find the weight of a sphere of lead 3·75 inches in diameter, the lead weighing 712 lb. per cubic foot.

99. Find the volume 1° of the greatest sphere, 2° of the greatest hemisphere, that can be cut out of a cube of wood measuring 7·5 inches on the edge.

100. Find the thickness of an 8-inch shell of half the weight of a solid ball of the same diameter and of like material.

101. If the atmosphere extend to a height of 45 miles above the earth's surface what ratio will its volume bear to that of the earth assumed to be a sphere of 7912 miles diameter?

102. A Stilton cheese is in the form of a cylinder, a Dutch cheese is in the form of a sphere. Find the length of the diameter of a Dutch cheese weighing 9 lb., a Stilton cheese 8 inches in diameter and 7 inches high weighing 6 lb.

103. Find the volume of the earth assuming it to be an oblate spheroid of 41,709,790 ft. polar axis and 41,852,404 ft. equatorial diameter.

104. On examining and taking the dimensions of a steep cistern which was supposed to be perfectly cylindrical, I found the bottom cross diameters to be 70 inches each, but the top diameters were 68 and 72 inches respectively. The depth of the vessel was 65 inches. What is the difference in the capacities of the true cylinder at 70 inches diameter and the one examined?

105. A steep cistern in the form of a frustum of an elliptic cone, the cross diameters at the bottom being 84 and 64 inches, and the diameters at the top 72 and 57 inches, is 50 inches deep, and is filled to the depth of 25 inches with dry barley. How many cubic inches does it contain?

106. A cylindrical iron tank, 20 feet long and 4 feet 6 inches in diameter, was placed horizontally on a flat car and filled with oil at Petrolia. When it arrived at Toronto, it was found upon being dipped from the top, to be 10 inches to the surface of the oil. What was the wantage in gallons?

ANSWERS.

N.B.—The answers to the mechanical drill problems and to the introductory questions are not given.

Exercise I.—(Page 11).—*1.* 190 cents. *2.* 781. *3.* 18,770. *4.* 83,035. *5.* 66. *6.* 53. *7.* 39. *8.* 109. *9.* 471. *10.* 114 cents. *11.* 115. *12.* 62. *13.* 1483. *14.* 84; 83. *15.* 146. *16.* 1197. *17.* 93 cents. *18.* 7; 63. *19.* $185. *20.* $475. *21.* 164 cents. *22.* 50. *23.* 74. *24.* 123. *25.* 112 cents; 125 cents. *26.* $72. *27.* $1077. *28.* $3156. *29.* $33,000. *30.* 478; $2555. *31.* 119. *32.* 155. *33.* 143. *34.* 103. *35.* $19,457; $2546; $2027; $24,030. *36.* 697. *37.* 113. *38.* 391 miles. *39.* 128. *40.* 56. *41.* 2916; 515. *42.* 47. *43.* 13 pounds; $6.31. *44.* 67; $15.90. *45.* 267. *46.* 4529. *47.* 3317. *48.* 25,529,955. *49.* 1189. *50.* 3143. *51.* 1,920,622. *52.* 62. *53.* 156. *54.* 106. *55.* 250. *56.* 122 yd. *57.* 326 mi.

Exercise II.—(Page 17).—*1.* 14 cents. *2.* 111. *3.* 28. *4.* 17. *5.* 8. *6.* 98. *7.* 8. *8.* 54. *9.* 14. *10.* 31,102; 15,188. *11.* 84. *12.* 53. *13.* 167. *14.* 39 cents. *15.* 72 cents. *16.* 26. *17.* $671. *18.* $2.98. *19.* 21. *20.* 125; 26. *21.* 65. *22.* 464. *23.* 22. *24.* Saturday, 945; 3055. *25.* 54; $235. *26.* 87; $2659. *27.* $679. *28.* 146. *29.* 14. *30.* 5. *31.* 50. *32.* 14. *33.* 59. *34.* 159 on whole. *35.* 17. *36.* $19. *37.* B, 76 bush.; A, $9. *38.* $977. *39.* $69,839. *40.* $5182. *41.* $1669. *42.* 11. *43.* 32. *44.* $4205. *45.* $242. *46.* 44. *47.* 53. *48.* $670. *49.* $4525. *50.* $1985. *51.* $114. *52.* 35. *53.* 29. *54.* Harry, 16 p. and 5 h.; Bessie, 9 p. and 9 h. *55.* 7; 19. *56.* 22 each. *57.* 28; Dick, 2. *58.* 9, 15; 50, 39, 43; 50, 48, 34; 2, 16. *59.* 3. *60.* 2. *61.* 13. *62.* 11, 8; 4, 3. *63.* 333. *64.* 19. *65.* 19. *66.* 20. *67.* 51. *68.* 225. *69.* 114. *70.* 84 years 85 days.

Exercise III.—(Page 24).—*1.* 132. *2.* 360. *3.* 1440. *4.* 10,080. *5.* 744. *6.* 8760. *7.* 51. *8.* 10. *9.* 108 miles. *10.* 7050. *11.* 36,900. *12.* 324. *13.* 224. *14.* 96. *15.* 48. *16.* 102. *17.* 592. *18.* 179,080. *19.* $1053. *20.* 972. *21.* 13,731. *22.* 1,063,842. *23.* $1776. *24.* $12.66. *25.* $399,000. *26.* 84,681 men. *27.* 7566. *28.* 1,855,230 copies. *29.* $131.43. *30.* $500.64. *31.* $36.54. *32.* $19,064. *33.* 9 cents. *34.* $4. *35.* 528,175 lbs. *36.* 106; 426. *37.* $3626. *38.* $1.74. *39.* $1.52. *40.* 244; 291. *41.* 85 cents.

42. $1.50. *43.* $12,859. *44.* 48. *45.* $6205. *46.* $495. *47.* $12.65. *48.* 8843; 406,778. *49.* 42; 84. *50.* 12; 348. *51.* 5226; 36,582. *52.* 540; 2160; $367.20. *53.* $137. *54.* $1053. *55.* $801. *56.* $12,648. *57.* $24.24. *58.* Gained $24. *59.* 35 years. *60.* 38 years. *61.* 972. *62.* 217,377. *63.* 296 yards, 432. *64.* 156.

Exercise IV.—(Page 29).—*1.* 14. *2.* 3 cents. *3.* 13 cts. *4.* 6. *5.* 7. *6.* 3 pounds. *7.* 36. *8.* $3. *9.* 29. *10.* $63. *11.* 701. *12.* $701. *13.* $2003. *14.* 94. *15.* $21.24. *16.* 3168. *17.* 27. *18.* 24 cents. *19.* $1.25. *20.* $1558. *21.* $19. *22.* 1007. *23.* 588. *24.* 330. *25.* 480. *26.* 550. *27.* $856. *28.* 12. *29.* 10. *30.* 3 cents. *31.* 5 brothers; 9 nuts. *32.* 16 to each boy; 20 to the girl. *33.* 11 trips; 107 passengers. *34.* 19 trips; 123 persons. *35.* 8 cents. *36.* 20 cents. *37.* 30 cents. *38.* 14. *39.* $64,043. *40.* 12. *41.* 4 cents. *42.* 6 hours. *43.* 8 seconds. *44.* 5 hours. *45.* 6 days 4 hours. *46.* 7 days.

Exercise VI.—(Page 37).—*1.* 800 ct. *2.* 9700 ct. *3.* 10,000 ct. *4.* 700,400 ct. *5.* 84 in. *6.* 12 ft. *7.* 154,000 lb. *8.* 672 hr. *9.* 33880 sq. yd. *10.* 32 pk. *11.* 192 oz. *12.* 108,864 cu. in. *13.* 608,000 oz. *14.* 15,840 ft. *15.* 192 pt. *16.* 40,320 min. *17.* 2880 sheets. *18.* 160,704 sq. in. *19.* 129,600″. *20.* 1504 qt. *21.* 2352 oz. *22.* 288 pt. *23.* 3072 cu. ft. *24.* 46,080 oz. *25.* 2,471,040 in. *26.* 847 ct. *27.* 7007 ct. *28.* 7 ct. *29.* 40,010 ct. *30.* 1192 oz. *31.* 1,434,407 oz. *32.* 252 pt. *33.* 86,164 sec. *34.* 31,556,929 sec. *35.* 2619′. *36.* 174 in. *37.* 36,240 sq. in. *38.* 639 cu. ft. *39.* $8.10. *40.* $11.96. *41.* $4857.60. *42.* $1,182,370. *43.* $988.20. *44.* 13909 yd. *45.* $45.60. *46.* $57.24. *47.* 317. *48.* 503. *49.* 44,640 min. *50.* 41,760 min. *51.* 5910. *52.* $10.46. *53.* $35.76. *54.* $3569.50. *55.* $21.12. *56.* 768. *57.* $207.60. *58.* 43,827,734. *59.* 48,400. *60.* 19,998. *61.* 12,410.

Exercise VII.—(Page 39).—*1.* 6 ft. *2.* 3 gal. *3.* 88 gal. 3 qt. 1 pt. *4.* $9.45. *5.* $16.02. *6.* $8.30. *7.* $70. *8.* $100. *9.* $40,10. *10.* 42 lb. 6 oz. *11.* 3 T. 1460 lb. *12.* 14 T. 1915 lb. 5 oz. *13.* 16 bu. 40 lb. *14.* 29 bu. 14 lb. *15.* 20 bu. 40 lb. *16.* 17 bu. 48 lb. *17.* 62 bu. 30 lb. *18.* 46 bu. 20 lb. *19.* 55 bu. 20 lb. *20.* 45 bu. 47 lb. *21.* 45 bu. 45 lb. *22.* 82 bu. 8 lb. *23.* 30 bu. 36 lb. *24.* 51 bu. 10 lb. *25.* 41 bu. *26.* 45 bu. 43 lb. *27.* 29 bu. 55 lb. *28.* 120 bu. 36 lb. *29.* 29 bu. 23 lb. *30.* 66 bu. 36 lb. *31.* 54 bu. 7 lb. *32.* 74 bu. 4 lb. *33.* 1687 lb. 8 oz. *34.* 10 T. 1504 lb. *35.* 674 T. 766 lb. 4 oz. *36.* 8 A. 1280 sq. yd. *37.* 30 gal. *38.* 36 gal. *39.* 31 T. 500 lb.

40. 3 mi. 720 yd. *41.* 1 mi. 55 yd.
42. 7926 mi. 1326 yd ; 7926 mi. 310 yd. *43.* 7899 mi. 744 yd. 2 ft.
44. 24,902 mi. 200 yd. *45.* 17 mi. 1247 yd. 2 ft. 8 in.
46. 3661 sq. yd. 7 sq. ft. *47.* 1 A. 308 sq. yd.
48. 1672 A. 448 sq. yd. 1 sq. ft. 48 sq. in.
49. At 10 hr. 46 min. 40 sec. p.m. on 12th March, 1893.
50. 63360 mi.; 1 sq. mi. *51.* 6 mi. 273 yd. 1 ft.

Exercise VIII.—(Page 41).—*1.* $93.41. *2.* 326 lb. 12 oz.
3. 74 T. 310 lb. *4.* 45 A. 639 sq. yd. *5.* 508 mi. 188 yd.
6. 140 bu. 3 pk. 1 gal. 3 qt. *7.* 84 T. 1830 lb. *8.* 67 gal. 2 qt.
9. 244 bu. 2 lb. *10.* 129 cords 3 cu. ft.
11. 1002 A. 1438 sq. yd. 7 sq. ft. 64 sq. in. *12.* 95 A. 146 sq. yd.
13. $84.09. *14.* 152 mi. 1210 yd. *15.* 1277 bu. 1 gal. 2 qt.
16. 22 mi. 1601 yd. *17.* 223 bu. 30 lb. *18.* 142 yd. 2 ft. 6 in.
19. 35 yd. 2 ft. 8 in. *20.* $2187.97. *21.* 108 mi. 1590 yd.
22. 105 A. 2904 sq. yd. *23.* 41 ft. 4 in. *24.* 160 sq. yd. 3 sq. ft.
25. 75 cu. ft. 951 cu. in.

Exercise IX.—(Page 43).—*1.* 14 T. 664 lb. *2.* 9 gal. 3 qt. 1 pt.
3. 3 T. 928 lb. 13 oz. *4.* 1467 bu. 55 lb. *5.* 2 mi. 1460 yd.
6. 32 lb. 11 oz. *7.* 9. *8.* A ; $12.91. *9.* 2 hr. 31 min. 17 sec.
10. 7 hr. 47 min. 55 sec. *11.* 659 bu. 58 lb. *12.* 84 mi. 382 yd. 8 in.
13. 8 gal. 2 qt. 1 pt. *14.* 201 bu. 3 qt. *15.* 13 cords 111 cu. ft.
16. 46 A. 3529 sq. yd.
17. 57 A. 381 sq. yd. cleared, and 29 A. 2606 sq. yd. woodland.
18. 9° 36′ 54″. *19.* 5° 6′ 52″ Lat.; 69° 50′ 39″ Long. *20.* 8° 58′ 31″.
21. 2 cords 80 cu. ft. *22.* 8 T. 1251 lb. *23.* $22.43. *24.* $322.53.
25. $77.98. *26.* 3750 lb.

Exercise X.—(Page 45).—*1.* 21 lb. 15 oz. *2.* 74 lb. 4 oz.
3. 17 gal. 2 qt. *4.* 33 ft. 6 in. *5.* 66 da. 19 hr. *6.* 1903 bu. 1 pk. 3 qt.
7. 3869 da. 18 hr. 36 min. *8.* 500 da. 21 hr. 51 min. 40 sec.
9. 175 mi. 1580 yd. *10.* 6 mi. 1623 yd. 2 ft. *11.* 52 cords 40 cu. ft.
12. 7 A. 4232 sq. yd. *13.* 6 pk. 1 gal. 3 qt. *14.* $6.32.
15. 2 mi. 753 yd. 1 ft. 6 in. *16.* 10 mi. *17.* 31 mi. 440 yd.
18. 2305 mi. 730 yd. *19.* 41 mi. 940 yd. *20.* 13 A. 38 sq. yd.
21. $28.44. *22.* $475.20. *23.* 911 gal. 1 qt. *24.* $4.53; the grocer.
25. 25 lb. *26.* 285 gal. 3 qt. *27.* 191 lb. 12 oz. *28.* 67 mi. 330 yd.
29. 64 ft. 2 in. *30.* 55 ft. 10 in. *31.* 48 bu. 18 lb.
32. 338 cords 56 cu. ft. *33.* $75.60. *34.* 15 min. 38 sec.
35. 792 yd. *36.* 1826 mi. 1510 yd.

Exercise XI.—(Page 47).—*1.* 3 lb. 6 oz. *2.* 2 lb. 7 oz. *3.* 3 T. 1539 lb. *4.* 3 T. 1675 lb. 10 oz. *5.* 4 gal. 3 qt. *6.* 4 gal. 2 qt. *7.* 93 bu. 1 gal. 3 qt. *8.* 47 bu. 1 pk. 1 gal. 2 qt. *9.* 4 da. 4 hr. 31 min. *10.* 4 hr. 46 min. 16 sec. *11.* 24′ 24″. *12.* $8.14. *13.* $45.75. *14.* 3 mi. 519 yd. *15.* 18 mi. 901 yd. 1 ft. 6 in. *16.* 21 mi. 945 yd. 8 in. *17.* 9 mi. 1537 yd. 2 ft. 6 in. *18.* 15 cu. yd. 21 cu. ft. 1152 cu. in. *19.* 11 cu. yd. 15 cu. ft. 960 cu. in. *20.* 6 A. 9058 sq. l. *21.* 3 A. 2061 sq. yd. *22.* 55 A. 1399 sq. yd. 3 sq. ft. *23.* 3 pk. 5 qt. 1 pt. *24.* 2 A. 968 sq. yd. *25.* 19 cu. ft. *26.* 5 cu. yd. 14 cu. ft. *27.* A, 48 bu. 30 lb.; B, 32 bu. 20 lb.; C, 16 bu. 10 lb. *28.* 11 A. 537 sq. yd. 7 sq. ft. *29.* 1 A. 3489 sq. yd. *30.* 2646 sq. yd. 7 sq. ft. 126 sq. in. *31.* 1 bu. 3 pk. 7 qt.

Exercise XII.—(Page 48).—*1.* 16. *2.* 14. *3.* 10. *4.* 211. *5.* 903. *6.* 42. *7.* 60. *8.* 8308. *9.* 576. *10.* 9. *11.* 154. *12.* 425. *13.* 9009. *14.* 10. *15.* 2112. *16.* 6360. *17.* 17,640. *18.* 97 and 6 mi. 476 yd. 2 in. *19.* 119. *20.* 107. *21.* 5021. *22.* 90,911. *23.* 5. *24.* 300. *25.* 7700. *26.* 33. *27.* 110 times. *28.* 152; 153. *29.* 118933. *30.* 26. *31.* 32 da. *32.* 218 da. and 500 lb. over. *33.* 3086. *34.* 210. *35.* 160. *36.* 36. *37.* 11 da. 1 hr. 21 min. 52 sec *38.* 360. *39.* 5 hr. 30 min. *40.* 14 hr. *41.* 225. *42.* 4404. *43.* 19 and 9 in. *44.* 84. *45.* 963. *46.* 12 da. *47.* 297. *48.* 234; 2 hr. 5 min. 30 sec.

Exercise XIII.—(Page 50).—*1.* 96 ct. *2.* $2.21. *3.* $3.90. *4.* $4.59. *5.* $1.33. *6.* $2.94. *7.* $14.44. *8.* $11.65. *9.* $9.80. *10.* $16.20. *11.* $2.16. *12.* $6.60. *13.* $2.52. *14.* $17.75. *15.* $32.45. *16.* $191.25. *17.* $8670. *18.* $198. *19.* $13,500. *20.* $1. *21.* $7750. *22.* $9245.25. *23.* $20.01. *24.* $24.01. *25.* $126.36. *26.* $21.09. *27.* $31.05. *28.* $108.81. *29.* $22.23. *30.* $37.17. *31.* $588.23. *32.* $20.79. *33.* $26.91. *34.* $386.46. *35.* $18.36. *36.* $27.72. *37.* $27.93. *38.* $450.31. *39.* $2.88. *40.* $4.93. *41.* $139.50. *42.* $9.72. *43.* $40.50. *44.* $44.75. *45.* $20.70. *46.* $79.80. *47.* $43.75. *48.* $428.64. *49.* $72.31. *50.* $3287.82. *51.* 3 ct. *52.* $4.39. *53.* $1.80; 36 (5 ct.). *54.* $5.63. *55.* 26 ct. *56.* 17 yd. *57.* 19 yd. *58.* 20 yd. *59.* 12 yd. *60.* 119 bu. *61.* 39 bu. *62.* 1320 lb. *63.* 27. *64.* 98. *65.* 55 ct. *66.* 6 lb. *67.* $10.03. *68.* 16 yr. *69.* 97 ct. *70.* 12 doz.

Exercise XIV.—(Page 55).—*1.* $67.17. *2.* $10.42. *3.* $87.75. *4.* $15. *5.* $26.75. *6.* $20.89. *7.* $3.23. *8.* $31.10. *9.* $14.19. *10.* $18.73. *11.* $23.76.

Exercise XV.—(Page 61).—*41.* 49 ct. *42.* 62 ct.
43. $576; 24 ct.; 1 ct. *44.* $50.92. *45.* 32 ct. *46.* $21; 42 ct.
47. 2 qt. *48.* 11 gal. *49.* 6 gal. *50.* 62,142 ft.; 12,359 ft.
51. 365 da. 5 hr. 49 min. 12 sec.; 365 da. 5 hr. 49 min. $5\frac{5}{11}$ sec.
52. Av., $2.50. *53.* Av., $8.50. *54.* $1.32; 44 ct.
55. $5.20; $1.30. *56.* $617. *57.* 1005 lb.; 201 lb. *58.* 3 in.

Exercise XVI.—(Page 63).—*1.* H. 14, E. 10. *2.* A. 11, J. 6.
3. R. 13, D. 7. *4.* W. 29, F. 47. *5.* 1st 15, 2nd 22.
6. 1st $2.75, 2nd $2.25 *7.* 1408 lb., 1232 lb. *8.* S. $5500, R. $4500.
9. C. $4885, H. $2885. *10.* 1st $3406, 2nd $4549. *11.* $213, $63.
12. $3572.50, $4372.50. *13.* $11.50, $7.50. *14.* 4 lb. 10 oz., 3 lb. 6 oz.
15. 17 gal. 3 qt., 13 gal. 3 qt.
16. 17 cords 24 cu. ft., 9 cords 104 cu. ft.
17. 52 yd. 1 ft. 6 in., 47 yd. 1 ft. 6 in. *18.* E, 11 ct., T, 7 ct., A 7, ct.
19. H. 14, A. 17, J. 17. *20.* 45 ct., 35 ct., 20 ct.
21. 92 lb., 106 lb., 122 lb. *22.* $138, $159, $123.
23. 12 yd., 22 yd., 10 yd. *24.* 13 qt., 10 qt., 17 qt.
25. 6 lb. 10 oz., 9 lb., 8 lb. 6 oz. *26.* 90 A., 70 A., 40 A.
27. 32 lb. 12 oz., 36 lb. 12 oz., 23 lb. 12 oz., 28 lb. 12 oz.
28. G. 15, B. 10. *29.* 36 ct., 27 ct. *30.* 16 ct., 8 ct. *31.* 75 ct., 25 ct.
32. $5.50, $16.50, $22. *33.* 4. *34.* 5. *35.* 7. *36.* 1225.
37. 12 oz. of green, 1 lb. 4 oz. of black. *38.* 48 lb., 64 lb., 96 lb.
39. $950. *40.* 28, 32, 40. *41.* 9. *42.* 63. *43.* G. 9, B. 6. *44.* 8.
45. 8.

Exercise XVII.—(Page 67).—*11.* 54′. *12.* 72′. *13.* 58′.
14. 51′ 7″. *15.* 70′ 10″. *16.* $280. *17.* $26.40. *18.* $44.55.
19. 1404. *20.* $53.46. *21.* 24″, 20″, 12″; 56″.

Exercise XVIII.—(Page 68).—*1.* 6. *2.* 6; 3 in. *3.* 9, 12 in.; 7, 6 in.
4. 39 yd. *5.* 41 yd. *6.* 22 yd.; 21 yd. *7.* 42 yd. either way.
8. $82.25. *9.* $58.50; $60.75. *10.* $173.80. *11.* 11 yd.
12. $19.55. (Or, *11.* 10 yd. 25 in. *12.* $19.17).

Exercise XIX.—(Page 69).—*1.* 12. *2.* 13. *3.* $9. *4.* $10.20.
5. $15.35.

Exercise XXII.—(Page 73).—*28.* 1200 A. *29.* 122 sq. in.
30. 112 sq. in. *31.* 2376 sq. ft. *32.* 620 sq. ft.
33. 815 sq. ft. 36 sq. in. *34.* 9′. *35.* 13′. *36.* 4′. *37.* 40 yd.
38. 48 yd. *39.* 18′ 6″. *40.* 24. *41.* 81. *42.* 192 sq. in.
43. $15,400.

Exercise XXIII.—(Page 74).—*1.* $32.12. *2.* $13.64. *3.* $22.66. *4.* $13.64. *5.* $7.26. *6.* 28. *7.* 11. *8.* 19. *9.* 11. *10.* 7. *11.* $82.10. *12.* $20.40. *13.* $38.40. *14.* 10. *15.* 10. *16.* 8. *17.* 34. *18.* 6480. *19.* 9000. *20.* 324. *21.* 900 sq. ft. *22.* 30. *23.* 5100. *24.* $30. *25.* $8772.50. *26.* 50. *27.* 16. *28.* 8. *29.* $7.74. *30.* $9. *31.* $20.24. *32.* $12.30. *33.* $18.

Exercise XXV.—(Page 79).—*15.* 12. *16.* $13\frac{1}{2}$. *17.* 6. *18.* 2. *19.* 6000 lb.; 600 gal. *20.* 150 bu.; 9150 lb. *21.* 60,016. *22.* 25,667. *23.* 44 cu. yd. *24.* 40. *25.* 15,000. *26.* 2520. *27.* 21,912. *28.* 252,450. *29.* 16. *30.* 30. *31.* 144. *32.* 2000. *33.* 6720. *34.* 1080, *35.* 640. *36.* 1600. *37.* 900. *38.* 432. *39.* 900. *40.* 3600. *41.* 3080 cu. yd. *42.* 125. *43.* $56. *44.* $33.25. *45.* $446.25. *46.* 1500. *47.* 91 bbl. 21 gal. *48.* 1392 lb. *49.* 13,500. *50.* 23 cords 80 cu. ft. *51.* $348.48. *52.* $12. *53.* $198. *54.* 1815. *55.* 19,360. *56.* 3520. *57.* 4 mi. *58.* 25′. *59.* 6′ 3″. *60.* 96′. *61.* 48′. *62.* 11′ 2″. *63.* 5′ 7″. *64.* 2′. *65.* 16′. *66.* 16′. *67.* 15″. *68.* 24′. *69.* 9′. *70.* 5′ 6″. *71.* 4′ 6″. *72.* 9′ 11″.

Exercise XXXV.—(Page 91).—*9.* 5. *10.* 325. *11.* 62. *12.* 11.

Exercise XLIX.—(Page 109).—*26.* $67\frac{1}{4}$ lb. *27.* $17\frac{7}{24}$ qt.

Exercise LVI.—(Page 115).—*13.* 47. *14.* $8\frac{3}{4}$ mi. *15.* 55 ct. *16.* $2.93. *17.* 31 ct. *18.* $2.04. *19.* $4.17. *21.* $751\frac{7}{8}$ lb. *22.* $7.19. *23.* $7027\frac{3}{8}$ lb.

Exercise LVIII.—(Page 117).—*25.* 36; $15\frac{3}{4}$ lb. *26.* $\frac{1}{1320}$ lb.; 9735. *27.* $9. *28.* 17 ct. per doz.; $21\frac{3}{7}$ ct. *29.* 19 lb. 2 oz. *30.* $76\frac{1}{2}$ lb. *31.* $393\frac{1}{4}$ lb.

Exercise LX.—(Page 119).—*1.* 60 oz. *2.* 1750 lb. *3.* 1564 yd. 1 ft. 4 in. *4.* 2258 sq. yd. 6 sq. ft. *5.* 5 cu. ft. 108 cu. in. *6.* 1 qt. 1 pt. *7.* 1 pk. $1\frac{1}{8}$ qt. *8.* 1 hr. 58 min. 48 sec. *9.* 3 lb. 12 oz. *10.* 1 T. $1428\frac{4}{7}$ lb. *11.* 3 mi. 97 yd. 2 ft. 4 in. *12.* 642 yd. *13.* 4463 sq. yd. 3 sq. ft. *14.* 180 da. $12\frac{7}{8}$ hr. *15.* 19 cords $13\frac{1}{2}$ cu. ft. *16.* 872 gal. $2\frac{5}{16}$ qt. *17.* 4 lb. 6 oz. *18.* 3 ft. $3\frac{5}{8}$ in. *19.* 1 bu. 2 pk. 1 qt. 1 pt. *20.* 9 A. $989\frac{1}{2}$ sq. yd. *21.* 1 lb. $10\frac{1}{4}$ oz. *22.* 1 yd. 2 ft. $9\frac{3}{7}$ in. *23.* 366 A. 293 sq. yd. 3 sq. ft. *24.* 11 hr. $22\frac{8}{17}$ min. *25.* 19 cords $64\frac{9}{10}$ cu. ft. *26.* 900 lb. *27.* 3 mi. $1447\frac{3}{11}$ yd. *28.* 14 A. 4640 sq. yd. *29.* 5 bu. $1\frac{1}{34}$ pk. *30.* 5 cu. ft. 1600 cu. in. *31.* 115 da. 11 hr. 12 min. *32.* $10. *33.* 17 lb. 1480 gr. *34.* $\frac{1}{4}$.

35. $2\frac{5}{16}$ lb. *36.* $\frac{1}{8}$. *37.* $\frac{23}{35}$. *38.* $\frac{7}{704}$. *39.* $3\frac{3}{704}\frac{977}{}$ mi. *40.* $7\frac{14}{1}\frac{45}{386}\frac{51}{}$.
41. $\frac{1}{40}$. *42.* $\frac{1}{180}$. *43.* $18\frac{7}{12}$ bu. *44.* $33\frac{3}{8}$ bu. *45.* $57\frac{14}{17}$ bu.
46. $26\frac{3}{8}$ bu. *47.* $34\frac{1}{8}$ bu. *48.* $30\frac{3}{8}$ bu. *49.* $30\frac{5}{8}$ bu. *50.* $22\frac{5}{8}$ bu.
51. $13\frac{1}{3}$. *52.* $3\frac{23}{35}$; *53.* $\frac{27}{4854}$. *54.* $\frac{27}{6470}$. *55.* 15. *56.* $\frac{1}{135}$. *57.* $\frac{6}{27}\frac{75}{000}$.
58. $32\frac{23}{89}$. *59.* $\frac{13}{64}$. *60.* $\frac{3}{8}$. *61.* $\frac{2}{65}$. *62.* $\frac{8}{88}$. *63.* $\frac{1}{20}\frac{35}{22}$. *64.* $\frac{2}{42}\frac{07}{}$.
65. $\frac{2}{70}\frac{01}{40}\frac{41}{}$. *66.* $\frac{1}{11}\frac{64}{25}$. *67.* $\frac{2}{4}\frac{01}{00}$. *68.* $\frac{3}{16}$. *69.* 320. *70.* 2592.
71. $11\frac{23}{78}$. *72.* $17\frac{13}{18}$. *73.* $1535\frac{5}{8}$ bottles. *74.* $\frac{14}{65}\frac{3}{}$. *75.* $\frac{23}{86}\frac{02}{}$.
76. $\frac{73}{73}$. *77.* $\frac{20}{86}\frac{03}{}$. *78.* $\frac{3}{4}$. *79.* $\frac{1}{20}$; $\frac{19}{20}$. *80.* $\frac{1}{8}$. *81.* $\frac{1}{4}$; $\frac{1}{8}$.
82. $\frac{18}{23}\frac{13}{00}\frac{3}{0}$; 2 T. $328\frac{14}{28}\frac{32}{78}$ lb. *83.* $207\frac{9}{22}$ bu. *84.* $\frac{19}{36}$; $\frac{13}{63}\frac{3}{}$. *85.* $\frac{9}{65}$.
86. $\frac{1}{6}$; $\frac{1}{4}$; $\frac{13}{45}$. *87.* 6 hr.; 4 hr. 48 min.; $\frac{13}{24}$. *88.* 13 ft. $1\frac{1}{2}$ in.
89. \$7.32; $\frac{61}{1}\frac{}{20}$. *90.* \$10.07; $\frac{12}{36}\frac{1}{0}$. *91.* $\frac{1}{8}$. *92.* $\frac{28}{86}\frac{1}{0}$. *93.* $\frac{15}{86}\frac{70}{00}\frac{9}{0}$.
94. $\frac{1}{18}$. *95.* $60\frac{3}{4}$ yr.; $\frac{1}{6}$. *96.* 18 mi. 60 rd. $3\frac{3}{4}$ yd. *97.* 25; $\frac{1}{4}$ in.
98. $69\frac{2}{1}\frac{0}{18}\frac{5}{8}$ mi. *99.* $68\frac{2}{11}$ mi. *100.* $\frac{1}{143}$. *101.* \$30,000.

Exercise LXI.—(Page 123).—*1.* 87 ct. *2.* 93 ct. *3.* \$1.58.
4. \$5.22. *5.* \$22.17. *6.* \$24.14. *7.* \$26.74. *8.* \$25.13.
9. \$19.63. *10.* \$2.74. *11.* \$15.16. *12.* \$5.49. *13.* \$54.11.
14. \$54.01. *15.* W.; \$5.69. *16.* \$46.64. *17.* \$258.82.
18. \$291.79. *19.* \$375. *20.* \$306.34. *21.* $91\frac{1}{24}$ ct. *22.* $29\frac{23}{35}$ in.
23. $27\frac{9}{7}\frac{0}{5}\frac{3}{0}\frac{}{1}$ in. *24.* $156\frac{4}{5}$. *25.* $333\frac{25}{48}$ gal.; $144\frac{23}{54}\frac{3}{0}\frac{3}{8}$ gal.
26. $27\frac{3}{8}$ @ 13 ct., $22\frac{2}{8}$ @ 18 ct. *27.* $22\frac{1}{2}$ lb.
28. 9 min. 25 sec. past 10 a.m. *29.* $25\frac{37}{63}$ mi.; $30\frac{33}{83}$ mi.
30. $3\frac{9}{14}$ mi.; $2\frac{9}{81}$ mi. *31.* $2\frac{7}{8}$ lb., $4\frac{5}{8}$ lb. *32.* $1\frac{17}{24}$ yd., $2\frac{5}{8}$ yd, $\frac{23}{24}$ yd.
33. 24 in., 40 in., 44 in.
34. A, $35\frac{95}{36}\frac{}{3}$ A.; B, $13\frac{27}{12}\frac{}{1}$ A.; C, $18\frac{2}{11}$ A.; D, $33\frac{1}{3}$ A. *35.* 492; 755.
36. 8(\$5) + 10(\$3.90). *37.* A, $39\frac{3}{8}$ bu.; B, $34\frac{13}{20}$ bu. *38.* $\frac{9}{20}$, $\frac{3}{10}$, $\frac{1}{4}$.
39. \$4.50 each. *40.* A, 4 mi.; B, $2\frac{9}{11}$ mi. per hr.
41. \$1462.22; \$1997.73; \$3990.05. *42.* \$19.58. *43.* $465\frac{9}{20}$ mi.
44. $54' \frac{7}{12}''$. *45.* $3746\frac{35}{63}$ sq. yd. *46.* $68\frac{3}{4}$ mi. *47.* 35 hr. 12 min.
48. $19\frac{9}{16}\frac{4}{74}\frac{97}{37}$ bu. *49.* $7837\frac{1}{27}$ sq.mi. *50.* $5\frac{44}{63}\frac{9}{}$ sq. yd. *51.* 1000.
52. $8\frac{1}{28}$. *53.* \$496.64. *54.* \$477.24. *55.* \$4942.27.
56. \$1357.71. *57.* \$2677.50. *58.* \$17.08. *59.* $16\frac{3}{7}$ ft.
60. $89\frac{23}{00}\frac{7}{}$ cu. in.; $[8\frac{3}{8}'' \times 4\frac{3}{20}'' \times 2\frac{1}{35}'']$. *61.* $65{,}406\frac{91}{144}$ T.
62. 6186 T. $1026\frac{37}{45}$ lb. *63.* $\frac{3}{16}$ in. *64.* $1{,}793{,}458\frac{1}{3}$ gal.
65. 1 hr. $28\frac{15}{20}\frac{91}{48}$ min. *66.* \$181.30. *67.* $19\frac{85}{36}\frac{87}{86}\frac{}{4}$ cu. ft.
68. \$3. *69.* A, \$8.80; B, \$12.10. *70.* $\frac{11}{15}$; \$1650. *71.* \$2.25.
72. $\frac{24}{35}$; A, \$4.50; B, \$4.80; C, \$7.20. *73.* 9 ct. *74.* \$5130.
75. 24 mi. to 25 mi. *76.* $63\frac{1}{33}$ mi.
77. 7.25 p.m. of 17th day; $7.47\frac{1}{2}$ p.m. *78.* $6\frac{5}{12}$ mi.; $12\frac{1}{12}$ mi.
79. $9\frac{3}{4}$ mi.; $5\frac{5}{12}$ mi.; $7\frac{7}{12}$ mi.; $2\frac{1}{6}$ mi. *80.* $3\frac{3}{4}$ sec.; $4\frac{2}{7}$ sec.; $3\frac{1}{3}$ sec.

81. (i) 15 sec.; (ii) $9\frac{3}{8}$ sec.; (iii) $37\frac{1}{2}$ sec.
82. $53\frac{5}{11}$ min.; 2 mi. 1576 yd. *83.* 2 min. $23\frac{9}{37}$ sec.
84. $\frac{1}{12}$, $\frac{1}{15}$, $\frac{3}{20}$; $6\frac{2}{3}$ da. *85.* $3\frac{1}{18}$ hr. *86.* 3 da. *87.* $67\frac{29}{45}$ da.
88. $\frac{15}{67}$ da.
89. 1st time, $\frac{2}{3}$ way round; A, $2\frac{2}{3}$ rounds; B, $1\frac{2}{3}$ rounds.
2nd " $\frac{1}{3}$ " A, $5\frac{1}{3}$ " B, $3\frac{1}{3}$ "
3rd " at starting point; A, 8 " B, 5 "

Together *Opposite.* *At Right Angles.*
90. 1 hr. $5\frac{5}{11}$ min. 12 hr, $32\frac{8}{11}$ min. 12 hr. $16\frac{4}{11}$ min. 12 hr. $49\frac{1}{11}$ min.
and at regular intervals of 1 hr. $5\frac{5}{11}$ min. thereafter.
91. 1 hr. 6 min., 2 hr. 12 min., 3 hr. 18 min., 4 hr. 24 min., 5 hr. 30 min., 6 hr. 36 min., 7 hr. 42 min., 8 hr. 48 min., 9 hr. 54 min.
92. $21\frac{7}{16}$ gal. *93.* $1\frac{29}{100}$ gal.

Exercise LXV.—(Page 136).—*10.* 78142·8571. *11.* 125.
12. 8275·8621. *13.* 3443·75. *14.* ·0089. *15.* ·0001. *16.* ·0009.

Exercise LXVI.—(Page 136).—*1.* 58 ft. 9 in.
2. 177·8889 + sq. yd. = 177 sq. yd. 8 sq. ft. *3.* 2689·39575 cu. ft.
4. 10,556. *5.* 5430 gal. *6.* 1255 lb. 1 oz. nearly.

Exercise LXIX.—(Page 138).—*1.* 3 T. 951 lb. 9·6 oz.
2. 4 yd. 9·45 in. *3.* 29 da. 12 hr. 44 min. 27·6 sec.
4. 100·9152 sec. *5.* 5488·56 sq. in. *6.* ·4738 mi. *7.* 1·76986 da.
8. 4·45 ch. *9.* 12·375 A. *10.* 17·85 A. *11.* 277·118 cu. in.
12. 8770·47 gal. *13.* 947·55 lb. *14.* 365 da. 5 hr. 48 min. 47·46 sec. [*Read* 365·242216 da. *for* 365·24216 da.]
16. 1609·33 in. *17.* 1093·623 yd. *18.* 2485·5 mi. *19.* 1550·03 sq. in.
20. 119·6 sq. yd. *21.* 4046·79 sq. m. *22.* 61·0255 cu. in.
23. 220·214 gal.

Exercise LXX.—(Page 139).—*1.* ·05. *2.* ·075. *3.* ·33$\frac{1}{3}$. *4.* 1.50.
5. ·005. *6.* 7%. *7.* 70%. *8.* 37$\frac{1}{2}$%. *9.* 225%. *10.* $\frac{3}{4}$%. *11.* \$21.
12. \$22.50. *13.* 150 yd. *14.* $5\frac{1}{2}$ lb. *15.* \$12.78$\frac{1}{8}$. *16.* \$·28$\frac{1}{10}$.
17. 6%. *18.* 3·2%. *19.* $\frac{1}{4}$%. *20.* 4%. *21.* 12$\frac{1}{2}$%. *22.* 2$\frac{1}{2}$%. *23.* $\frac{1}{2}$.
24. $\frac{1}{5}$. *25.* $\frac{1}{8}$. *26.* $\frac{1}{3}$. *27.* $\frac{1}{8}$. *28.* \$243. *29.* \$338.40. *30.* 1242 gal.
31. 16. *32.* \$220 per annum. *33.* \$675. *34.* 88. *35.* 110 lb.
36. $11\frac{7}{11}$%. *37.* 14%. *38.* 4, 11, 33, 30, 16, 5 and 1% respectively.
39. \$127·50. *40.* \$9·07. *41.* \$7·40. *42.* \$4030. *43.* $7\frac{9}{13}$%.
44. $77\frac{7}{9}$%.

Exercise LXXI.—(Page 141).—*1.* 25% G. *2.* 20% G. *3.* 12% L. *4.* $13\frac{3}{5}$% G. *5.* $46\frac{2}{3}$% L. *6.* 1 G. *7.* \$9. *8.* \$11.25. *9.* \$49.50. *10.* \$4·25. *11.* \$3·75. *12.* \$54. *13.* 1·06; 1·11; 1·20; 1·075; $1·33\frac{1}{3}$; 2·10. *14.* ·96; ·93; ·9; ·925; $·66\frac{2}{3} = \frac{2}{3}$; $·96\frac{2}{3}$. *15.* \$16. *16.* \$28·50. *17.* \$1664. *18.* \$16. *19.* \$57. *20.* \$1100. *21.* \$46·92; \$6·12. *22.* \$1·92, 80 ct. *23.* \$5; $16\frac{2}{3}$%. *24.* \$1209. *25.* \$1320; \$2280; $72\frac{8}{11}$%. *26.* 75%. *27.* \$1191·68; $32\frac{81}{102}$%. *28.* 50%.

Exercise LXXII.—(Page 144).—*1.* \$131·25. *2.* \$471·25. *3.* \$232·47. *4.* \$332·50. *5.* \$1680; \$1120; \$700. *6.* \$4500, \$3600, \$2400. *7.* \$82·50. *8.* 0·8%. *9.* \$187·50; $1\frac{7}{8}$%. *10.* \$1980, \$2640. *11.* \$2393·62, \$5106·38. *12.* $1\frac{1}{8}$%. *13.* $\frac{7}{8}$%; $87\frac{1}{2}$ ct. per \$100. *14.* \$46·87$\frac{1}{2}$. *15.* $\frac{3}{4}$%. *16.* \$2000. *17.* \$12500. *18.* \$245. *19.* \$88333·33, \$14000. *20.* \$1851·94. *21.* \$4273·50. *22.* \$18918. *23.* 65 ct. per \$100. *24.* \$5930. *25.* \$32350·89, \$19410·54, \$16175·45, \$22645·62, \$22645·62. [*Read* \$175000 *for* \$75000]. *26.* \$52000, \$44250.

Exercise LXXIII.—(Page 147).—*1.* \$22·50; \$7·50; \$33·75; \$3·75; \$6·56 *2.* \$135; \$84·38; \$50·63; \$47·25. *3.* \$4797·50; \$4868·75; \$4908·33; \$4827·19. *4.* \$457·47. *5.* $2\frac{1}{4}$%. *6.* $2\frac{1}{8}$%. *7.* \$249.15. *8.* \$5100; \$5050; \$5025·37. *9.* \$2. *10.* 5%. *11.* \$61200. *12.* \$1388·62. *13.* \$288·39. *14.* \$18909·18. *15.* 6652 lb., 1901 lb. *16.* \$25663·44. *17.* \$9653·38.

Exercise LXXIV.—(Page 150).—*1.* \$374. *2.* \$510. *3.* \$176·80. *4.* \$166·50. *5.* \$230·85. *6.* \$80·66. *7.* \$206·60. *8.* \$284. *9.* \$25. *10.* 40%. *11.* \$58·70; 48%. *12.* 81%. *13.* \$4926·95; \$147·23; \$62·37; $73\frac{1}{2}$% nearly. *14.* $2\frac{1}{2}$% loss. *15.* Nothing. *16.* 16,348 bu.; 16,327 bu. *17.* \$2400.

Exercise LXXV.—(Page 155).—

	Date of Maturity.	*Term of Discount.*	*Discount.*	*Proceeds.*
1.	4 Sept., 1886	91 da.	\$3·74	\$246·26
2.	27 June, 1885	26 "	2·34	467·66
3.	17 Dec., 1883	85 "	2·18	185·32
4.	31 May, 1887	88 "	·99	67·76
5.	31 Mar., 1888	58 "	12·50	970·88

6. \$291·94. *7.* \$698·84. *8.* \$336·31.

No.	*Days to run.*	*Interest.*	*Exchange.*	*Proceeds.*
9. 1	62	$5·30	$0·89	$439·41
2	74	2·13	·30	147·37
3	100	5·09	·53	259·68
4	114	11·24	1·03	501·82
5	122	9·13	·78	380·43
				$1728·71.

10. Proceeds: $258·20; $112·07; $438·05; $53·22; $94·72; $313·32. Total, $1269·58. *11.* 7½%.

Exercise LXXVI.—(Page 159).—*1.* $18. *2.* $32·25. *3.* $55·60. *4.* $15·10. *5.* $12·39. *6.* $5·28. *7.* $5·73. *8.* $3·35. *9.* $6·17. *10.* $809. *11.* $567.82. *12.* 6%. *13.* 7%. *14.* 5%. *15.* 6%. *16.* 1 yr. 276 da. *17.* 128 da. *18.* Oct. 13. *19.* 20 yr. *20.* 25 yr. *21.* 8%; 8·034%. *22.* $7·27; $7·14. *23.* 11⅛%. *24.* 10·267%.

Exercise LXXVII.—(Page 162).—*1.* $926·10; $126·10. *2.* $497·19; $72·19. *3.* $281·38; $31·38. *4.* $404·83; $38·16. *5.* $766·95; $44·45. *6.* $1·06. *7.* $1·0609. *8.* $1·061364. *9.* 1·157625. *10.* 1·276281, *11.* 1·4071. *12.* 1·040604. *13.* 1·082857. *14.* 1·126825.

Exercise LXXVIII.—(Page 163).—*1.* $22·58. *2.* $2364·33. *3.* $71·41.

Exercise LXXIX.—(Page 165),—*1.* 12 Nov. *2.* 17 Dec. *3.* 30 June. *4.* 9 mos. *5.* 10 Oct. *6.* 2 Nov. *7.* 14 Sep. *8.* 8 July.

Exercise LXXX.—(Page 170).—*1.* $3275. *2.* $1237·50. *3.* $32700. *4.* $10468·75. *5.* $69231·25. *6.* $6792·04. *7.* $182062·50. *8.* $111000. *9.* $22812·50. *10.* $712·50. *11.* $4200. *12.* $210. *13.* 3·24%. *14.* 125 : 126. *15.* 110¼. *16.* 117⅛. *17.* 90⅝. *18.* 280. *19.* $15807·63. *20.* $19989; $827·50; 4·14%. *21.* $18000; $840. *22.* $160000. *23.* $35038·44. *24.* $3114·14. *25.* 102¾.

Exercise LXXXI.—(Page 173).—*1.* $656·25, $1093·75. *2.* $1070·30, $789, $1296·70. *3.* $18004·83, $13495.17. *4.* $20419·64, $34486·50. *5.* $13138·05, $10019·95. *6.* $8800, $10400. *7.* $11000; $432, $576. *8.* $16434·78, $24260·87. *9.* $185·81, $126·99, $121·40. *10.* $2216·98, $2480·08, $3152·94. *11.* $9368·20, $10021·80. *12.* $13000, $8000. *13.* $935065, $545455, $519480.

Exercise LXXXII.—(Page 180).—*1.* Between 41,709,500 ft. and 41,710,500 ft. $\frac{1}{800}$. *2.* $\frac{1}{7900}$.
3. To 5 sig. fig., 13·596. *4.* To 4 sig. fig., 13·60, $\frac{1}{2700}$.
5. 9·37505. *6.* 0·301030. *7.* 9·54010. *8.* 579·582. *9.* 0·273638.
10. 2674650, *11.* 62·4260. *12.* 10. *13.* 4301. *14.* 76·58. *15.* 607.
16. 700. *17.* 0·352644. *18.* 49·9999. *19.* 632291.
20. 213,526,000,000. *21.* 2·53998. *22.* 4·53593. *23.* 11109·7.
24. 0·000,000,318,310. *25.* 0·000,000,023,025,9. *26.* 1·77828.
27. 1·09362. *28.* 1·60933. *29.* 0·404678. *30.* 0·880860.
31. 1907·1854 Kg. *32.* 1016·048 Kg. *33.* 1609·33 m.
34. 119·601 sq. yd. *35.* 40·4679 a. *36.* 28·316 l. *37.* 4·541028 l.
38. 62.356 lb. *39.* 277·118 cu. in. *40.* 41,773,300 ft.

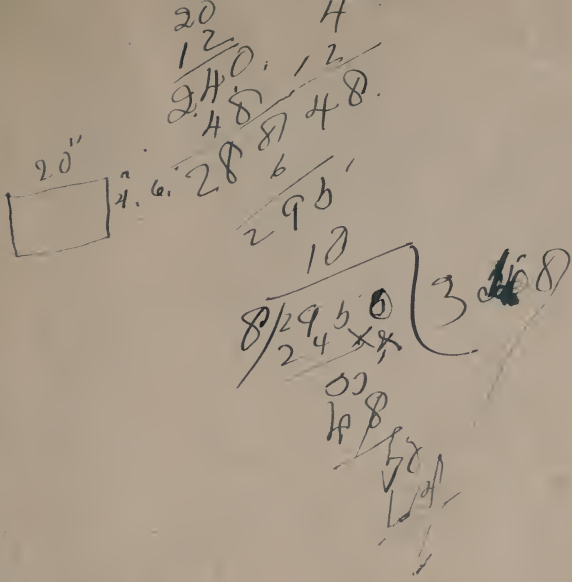
Exercise LXXXIII.—(Page 182).—*35.* $2^4 \times 3^2 \times 5$.
36. $2^3 \times 3^3 \times 5^2 \times 13$. *37.* $2^3 \times 7^2 \times 29$. *38.* 2^{10}.
39. $2^3 \times 3^6 \times 7 \times 11 \times 13$. *64.* 0·25. *65.* 0·2. *66.* 0·1.
71. $\$1 \times 1{\cdot}05^3$. *72.* $\$1 \times 1{\cdot}025^6$. *73.* $\$1 \times 1{\cdot}0125^{12}$.
75. $\$1000 \div 1{\cdot}04^5$. *76.* $\$1000 \div 1{\cdot}02^{15}$. *77.* $\$1000 \div 1{\cdot}01^{22}$.
78. $31·25. *79.* $25. *80.* $27·06. *81.* $27·35.
82. 3rd, $\$100(1{\cdot}03^2 + 1{\cdot}03 + 1)$;
6th, $\$100(1{\cdot}03^5 + 1{\cdot}03^4 + 1{\cdot}03^3 + 1{\cdot}03^2 + 1{\cdot}03 + 1)$;
$309·09, $418·36, $530·91, $646·84. *83.* $923.

Exercise LXXXV.—(Page 186).—*1.* 22. *2.* 99. *3.* 43.
4. 321. *5.* 32·1. *6.* 3·21. *7.* 48·18. *8.* 0·07097. *9.* 0·2646.

Exercise LXXXVI.—(Page 187).—*1.* 23456. *2.* 35·791.
3. 2·7313. *4.* 8·6308. *5.* 0·55955. *6.* 0·24900. *7.* 2·2361.
8. 22·361. *9.* 223·61. *10.* 0·22361. *11.* 0·022361. *12.* 7·0711.
13. 70·711. *14.* 707·11. *15.* 0·70711. *16.* 0·070711.
17. 0·0070711. *18.* 0·63246. *19.* 4·4721. *20.* 13·416. *21.* 8·9443.
22. 26·814. *23.* 1·1180. *24.* 0·63888. *25.* 0·86603. *26.* 0·79057.
27. 0·79772. *38.* 2·15. *39.* 6·04. *40.* 6·42.

Exercise LXXXVII.—(Page 190).—*1.* 33′ 7·2″.
2. 62′ 8·625″, 80′ 8·625″. *3.* 0·2″. *4.* 8′ 2·182″. *5.* 5081 yd.
6. $384·38. *7.* 384 sq. in., 315·375 sq. in. *8.* 36 ft.
9. 58·9 ft., 92·57 ft. *10.* 31·7495 A. *11.* 53 sq. ft., 63 sq. ft.
12. 551 sq. ft., 1653 sq. ft., 2755 sq. ft.
13. 4·63392 A., 4·80096 A., 4·968 A., 5·13504 A., 5·30208 A.
14. 146 yd. 2 ft. *15.* 8·415978 A. *16.* 1549 gal. *17.* 1 in.
18. 12 in. *19.* 68·921 cu. in. *20.* 87·72 sq. in. *21.* 335·41 cu. in.

22. 2·211 in. *23.* 649·52 cu. in. *24.* 5·4 in. *25.* 6·8457 in. square.
26. 645·225 lb. *27.* 0·000,006,7 in. *28.* 236·32 lb. *29.* 584·59 lb
30. 5066 cu. in. *31.* 50 cu. ft. 288 cu. in.
32. 117,333,333⅓ cu. yd. *33.* 159·25 cu. yd. *34.* 406 gal.
35. 394·4 gal. *36.* 11840 cu. ft.
37. 3464344 cu. yd., 3096908 cu. yd. *38.* 1492·36 cu. ft.
39. 30·484 cu. ft. *40.* 5128·27 gal. *41.* 5776·71 lb. *42.* 7008 lb.
34. 244·369 cu. ft. *44.* 771 cu. in. *45.* 32·64 cu. in., 37·44 cu. in.
46. 64⅓ cu. ft. *47.* 13954·3 lb. *48.* 905 in. *49.* 17 in.
50. 20·75 in. *51.* 3·464 in. *52.* 26·7 in., 24·4 in., 12·5 in.
53. 15 ft. *54.* 4·29 in., 8·8 in., 23·4 in. *55.* 60 sq. yd.
56. 60 sq. yd. *57.* 2·9274 sq. ch. *58.* 166·417 A. *59.* $260653.
60. 16672·5 sq. ft. *61.* 13 mi. 1200 yd. *62.* 20·5 in.
63. 410 yd. 1 ft. 5 in. *64.* 66579 mi. *65.* $138\frac{1}{16}$. *66.* $99\frac{187}{192}$ mi.
67. $54\frac{11}{36}$. *68.* $2095\frac{5}{21}$ ft. *69.* 43,827,033 yd., 43,827,735 yd.
70. 1·093827 yd. *71.* $19\frac{9}{14}$ sq. in. *72.* $\frac{11}{16}$ sq. in.
73. 829·58 sq. ft. *74.* $\frac{121}{360}$ sq. in. *75.* 386·146 sq. ft.
76. 0·615 sq. ft., 1·133 sq. ft. *77.* $96·10 *78.* 92·88 sq. in.
79. 62·832 sq. ft. *80.* 1269·21 sq. ft. *81.* 40 sq. ft. 48 sq. in.
82. 19·37 sq. ft. *83.* 49 sq. ft. 120·6 sq. in.
84. 52 sq. ft. 134·7 sq. in. *85.* 17 sq. ft. 96·69 sq. in. *86.* 41 yd.
87. 113·1 sq. in. *88.* 45·837 sq. in. *89.* 196,940,000 sq. mi.
90. 628·32 cu. in. *91.* 693 cu. in. *92.* $5\frac{15}{28}$ gal. *93.* 13·89 mi.
94. 15·783 mi. *95.* 0·1502 in. *96.* 16889·24 cu. in.
97. 5·5096 gal. *98.* 11·377 lb. *99.* 110·446 cu. in. *100.* 0·825 in.
101. 153 to 1000. *102.* 10·01 in. *103.* $2·598817 \text{ cu. mi.} \times 10^{11}$.
104. 68·068 cu. in. *105.* 99041 cu. in. *106.* 253 gal.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